SOLUTION OF THE PROBLEM OF CRITICAL PATH'S FINDING IN EXCEL ON THE BASIS OF REDUCING IT TO ORDINARY TRANSPORTATION TASK

Ludmila Malyaretz[1], Oleksandr Dorokhov[1], Mimo Drašković[2]

[1]Kharkiv National University of Economics
Kharkiv, Ukraine. E-mail: aleks.dorokhov@meta.ua

[2]University of Montenegro
Kotor, Montenegro. E-mail: veso-mimo@t-com.me

The problem of determining the critical path for the scheduling tasks has been considered. An original technique for practical calculations using the available Solver add-on in Excel has been described. The proposed approach is based on reducing the task to solving the ordinary transportation problem, in particular to the problem of finding the longest path. Examples of solutions with the test input data and corresponding screenshots are given. The practical steps of the user's action in the process of direct solution in Excel are consistently described. The analysis of the results of the proposed method has been performed and presented. It is established that presented method for calculating the critical path requires minimal efforts from the users, regardless of the dimension of the tasks.

Keywords: transportation problem, definition of the critical path, add-on for finding a solution in Excel

1. Introduction

As it is well known, there are many practical problems, including transport character and project management which are formulated and solved with use of network models.

Almost in each manual on economic-mathematical methods and models mathematical bases of network planning and management [1-6] are stated.

Necessity of development of effective ways of planning of complex processes has led to creation the essentially new methods of network planning and management.

More often for construction of network models five basic algorithms are used: findings of minimal tree; findings of the shortest way; definitions of the maximal stream; minimization of cost of a stream in a network with the limited throughput; findings of a critical path (way) [6].

Thus the algorithm of a critical path is the most known method in planning, drawing up of time schedules and managements of projects.

2. Description of the Researches

The main, basic problem in calendar network planning of manufacture is definition of "a critical path". It represents sequence of the operations which are not having a reserve, a stock of time.

Operation is considered as critical if the delay of its beginning leads to increase in a termination date of all process (part of which is considered operation) as a whole.

2.1. Graphic representation of the network schedule

In case of graphic representation of the network schedule, its arrows (focused arches) represent the certain operations. The figure near each arrow means duration of corresponding operation. Initial and final points of any operation correspond to meeting events (initial and final). The operations which are starting with some event cannot begin while the operations entering into this event will not be completed yet all.

"The critical path (way)" on the network schedule represents the continuous chain of operations connecting initial event of a network with finishing.

The purpose of our work is reduction of a problem of search of a critical path to usual transport task, namely a task of search of the longest way.

By analogy to a transport task we shall consider units of the network schedule (except for initial and final), as transit points. Certainly, thus for a critical way the requirement that it is possible to arrive to each transit point only from one previous point and to go only to one subsequent point is carried out.

Earlier in work of authors [5] the task of finding of the shortest way by **"Solver"** tool in spreadsheet **Excel** has been examined.

The offered approach can be applied to calculation of a critical path. For an illustration of the offered approach we shall consider an example of the network schedule (Fig. 1) from work of Hemdi A. Taha [4].
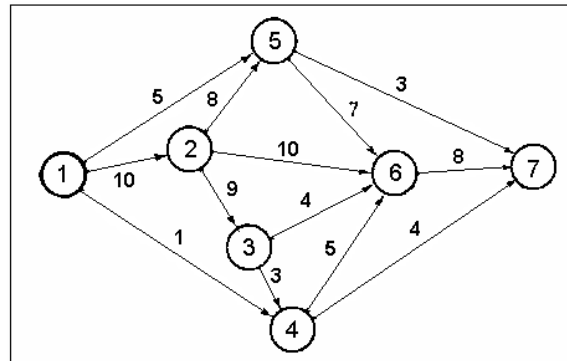

Figure 1. The network schedule for a test example

It is necessary to emphasize, that as a whole the idea of use of methods of linear programming for definition of a critical path expressed and earlier, but its computer realization in this case is important. In fact practical network models can be much more difficult than simple graph, represented on Fig. 1.

Therefore it is necessary to organize calculations so that it was feasible for the usual user. Usually in real network model there are some tens events (points). Accordingly, it is necessary to fill tables of initial data of the big size with dimension in some tens elements. There is a problem to reduce this work up to a possible minimum and to receive thus the optimum decision.

2.2. Technique and the order of practical calculations

Let's consider an offered technique and practical actions for an example of graph, shown on Fig. 1.

Firstly, we shall enter in **Excel** corresponding with the network schedule (Fig. 1) data for durations of works t_{ij} between each pair of points $T_i - T_j$ (Fig. 2).

	I	J	K	L	M	N	O
4	Duration of works between each pair of points						
5	t_{ij}	T_2	T_3	T_4	T_5	T_6	T_7
6	T_1	10	−100	1	5	−100	−100
7	T_2	0	9	−100	8	10	−100
8	T_3	−100	0	3	−100	4	−100
9	T_4	−100	−100	0	−100	5	4
10	T_5	−100	−100	−100	0	7	3
11	T_6	−100	−100	−100	−100	0	8

Figure 2. Input data for the test example

All points, except for last point T_7, we will consider as points of departure. They are listed in the left column of the table t_{ij}. All points, except for start point T_1, we will count as points of destinations. They are listed in the top line of the table t_{ij}. Transit points $T_2 - T_6$ are considered both as points of departure and as points of destinations.

The durations of works between identical transit points $T_k - T_k$ are equal to zero. Between some points there are no communications, therefore we set the corresponding duration of works equal to very big negative number ($t_{ij} = -100$), that by search critical (the longest way) these forbidden transitions automatically were rejected.

Let's describe practical steps for filling the given table. To exclude from consideration fictitious durations on the forbidden transitions, it is expedient to represent (by means of conditional formatting) it's by their grey colour on a grey background. For this purpose we bring in any free cell number *-100* (duration for the forbidden transitions) and copy it in the buffer of an exchange.

Then, keeping pressed key *Ctrl*, we allocate by the mouse the table t_{ij} (without headings) and insert contents from the buffer of an exchange (at once into all cells of the table). Further (not removing allocation) in menu *Format* we use item *Conditional formatting*. On the panel of conditional formatting we fill a field *Condition 1: Cell Value Is* and *less than* by *-99*. We press the button *Format* and on panel *Format Cells* (item *Font*) we set colour of numbers, and on item *Patterns* - colour of a background. After that we enter real duration of works for all possible transitions between points.

Similarly zero duration of works between pairs of identical transit points also can be entered by one operation. For this purpose it is necessary to copy a cell with value *0* on the buffer of an exchange. Further it is necessary to allocate by mouse (with pressed key *Ctrl*) the cells on diagonal $T_k - T_k$ and to insert *0* from the buffer of an exchange at once into all allocated cells. By the described actions work on data input is shown up to a necessary minimum.

In the following table (the same size) for x_{ij} we shall define transitions between points (Fig. 3). If between points $T_i - T_j$ there is no transition then we accepted $x_{ij} = 0$ and if transition is exist then $x_{ij} = 1$. First we fill all cells of the table x_{ij} with 1 (all $x_{ij} = 1$). Naturally, they are written into all cells of the table by one operation (by copying from buffer of an exchange at once in all the allocated cells).

	I	J	K	L	M	N	O	P
13				*Transitions between points*				
14	x_{ij}	T_2	T_3	T_4	T_5	T_6	T_7	*Sum 1*
15	T_1	1	1	1	1	1	1	6
16	T_2	1	1	1	1	1	1	6
17	T_3	1	1	1	1	1	1	6
18	T_4	1	1	1	1	1	1	6
19	T_5	1	1	1	1	1	1	6
20	T_6	1	1	1	1	1	1	6
21	*Sum 2*	6	6	6	6	6	6	36

Figure 3. Transitions between points start (initial) position

However, as follows from a condition of a problem, the critical path passes through transit points only once. Therefore in each line and in each column of the table x_{ij} for a critical path there should be only one *1* (similarly a task about destinations). Therefore in the table x_{ij} are added final right column and below line in which formulas of summation (by function *SUM*) are entered.

For this purpose it is necessary to allocate the table x_{ij} without headings, but with additional right column and below line, and to press on tools panel the auto summa button *Σ*. Then in all cells of an additional column and an additional line will be automatically written down formulas of summation. Certainly, for a critical path all these sums should be equalled to *1*.

On an empty place of spreadsheet *Excel* for calculation of critical path duration we write formula *SUMPRODUCT* (*Range t_{ij} ; Range t_{ij}*). At first, before calculation's start, this duration is equal *-1723* (Fig. 4).

	K	L	M
22	*Duration of a critical path*		
23		*Max*	
24	*Function of the purpose*	*−1723*	

Figure 4. Length of a critical path

2.3. The finding of the critical path

Now we shall go directly to search of a critical path. We put the table cursor on a target cell and through the menu *Tools* call "*Solver* " Add-In (Fig. 5).
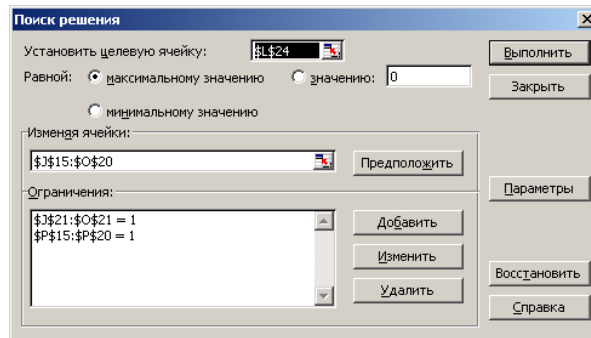
Figure 5. Window with the parameters of decision search in Solver Add-In

On the **Solver** panel window we set **Target Cell** equal **Maximal value**. In a field **Changing cells** we specify a range x_{ij}.

We set two restrictions: **Sum1** on table lines and **Sum2** on table columns should be equal **1**.

Then we press the button **Parameters** and put flags **Linear model** and **Non-negative values**.

Finally we press the button **To execute** and have received the optimum decision.

In the transformed table of transitions (Fig. 6) now in each line and in each column is only one **1** unit, all other numbers are zero.

	I	J	K	L	M	N	O	P
13				**Transitions between points**				
14	x_{ij}	T_2	T_3	T_4	T_5	T_6	T_7	**Sum 1**
15	T_1	**1**	0	0	0	0	0	**1**
16	T_2	0	**1**	0	0	0	0	**1**
17	T_3	0	0	**1**	0	0	0	**1**
18	T_4	0	0	0	0	**1**	0	**1**
19	T_5	0	0	0	**1**	0	0	**1**
20	T_6	0	0	0	0	0	**1**	**1**
21	**Sum 2**	**1**	**1**	**1**	**1**	**1**	**1**	**6**

Figure 6. The received decision

For simplification of results visualization it is expediently to allocate all zero in the table so that they did not prevent to see a critical path.

For example, by means of conditional formatting it is possible to show zero (numbers, smaller then **0,01**) as a grey colour on grey background. Numbers on diagonal $T_k - T_k$ also do not have any helpful information. Therefore it is possible to set a grey background for these cells too for what it is necessary to click mouse (at pressed key **Ctrl**) on diagonal cells and to set a demanded background at once for all of them.

Now on Fig. 6 only the critical path is allocated. From initial point T_1 there is a transition to point T_2. From point T_2 there is a transition to T_3. Further from point T_3 there is a transition to T_4, and from point T_4 there is a transition to T_6. At last, from point T_6 there exists transition at once to finish point T_7. The critical path does not take place through point T_5, therefore in the optimum decision fictitious transition from T_5 to T_5 is specified.

On Fig. 7 found critical path $T_1 \rightarrow T_2 \rightarrow T_3 \rightarrow T_4 \rightarrow T_6 \rightarrow T_7$ is represented.
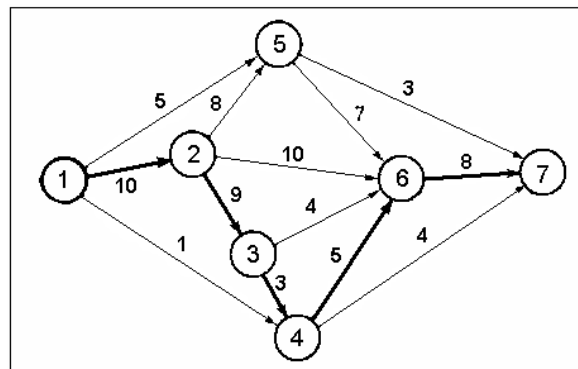

Figure 7. Found critical path

Duration of a critical path is equal *10+9+3+5+8=35*. As clear from Fig. 3, there is a reserve of time in *21* unit for performance of work *1-4*; *4* units for work *3-6; 7* units for *2-6*; in the sum of *2* units for works *2-5* and *5-6*; in the sum of *27* units for works *1-5* and *5-7*.

2.4. The example for more difficult network schedule

Let's shortly examine one more example of calculation of a critical path for the network schedule represented on Fig. 8. The filled corresponding table for work's durations is shown on Fig. 9.
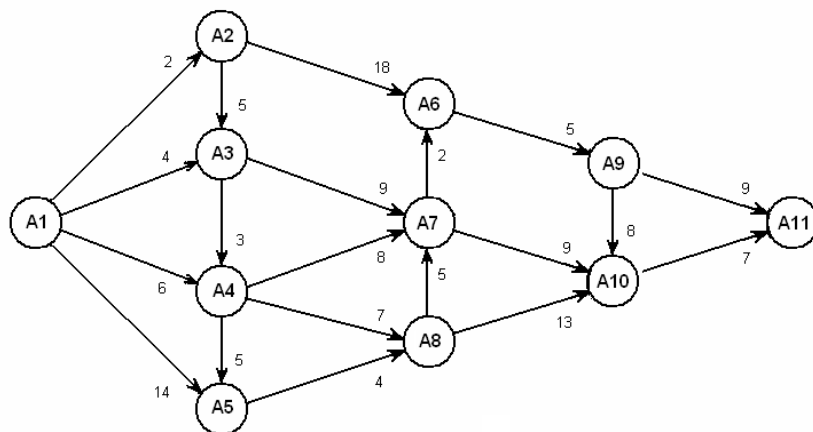

Figure 8. The network schedule for a test example of greater dimension

t_{ij}	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}
A_1	2	4	6	14	−100	−100	−100	−100	−100	−100
A_2	0	5	−100	−100	18	−100	−100	−100	−100	−100
A_3	−100	0	3	−100	−100	9	−100	−100	−100	−100
A_4	−100	−100	0	5	−100	8	7	−100	−100	−100
A_5	−100	−100	−100	0	−100	−100	4	−100	−100	−100
A_6	−100	−100	−100	−100	0	−100	−100	5	−100	−100
A_7	−100	−100	−100	−100	2	0	−100	−100	9	−100
A_8	−100	−100	−100	−100	−100	5	0	−100	13	−100
A_9	−100	−100	−100	−100	−100	−100	−100	0	8	9
A_{10}	−100	−100	−100	−100	−100	−100	−100	−100	0	7

Figure 9. Duration of works for a test example of greater dimension

The size of the second task is larger than the previous one. Data, which are necessary for typing manually, make a small part of the table (for this example nearly 20%).

The optimum decision was received in the form (see Fig. 10) where owing to conditional formatting the critical path is allocated. We write out it from the lines (Fig. 10): $A_1{\rightarrow}A_2{\rightarrow}A_3{\rightarrow}A_4{\rightarrow}A_5{\rightarrow}A_8{\rightarrow}A_7{\rightarrow}A_6{\rightarrow}A_9{\rightarrow}A_{10}{\rightarrow}A_{11}$. The summary (total) duration of works on a critical path is equal *2+5+3+5+4+5+2+5+8+7=46* time units.

x_{ij}	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}	Sum 1
A_1	1	0	0	0	0	0	0	0	0	0	1
A_2	0	1	0	0	0	0	0	0	0	0	1
A_3	0	0	1	0	0	0	0	0	0	0	1
A_4	0	0	0	1	0	0	0	0	0	0	1
A_5	0	0	0	0	0	0	1	0	0	0	1
A_6	0	0	0	0	0	−100	−100	1	0	0	1
A_7	0	0	0	0	1	0	0	0	0	0	1
A_8	0	0	0	0	0	1	0	0	0	0	1
A_9	0	0	0	0	0	0	0	0	1	0	1
A_{10}	0	0	0	0	0	0	0	−100	0	1	1
Sum 2	1	1	1	1	1	1	1	1	1	1	10

Figure 10. The final decision for a test example of greater dimension

On Fig. 11 the found critical way on the network schedule is represented. Numbers in brackets near arrows show reserves of time for performance of noncritical operations.
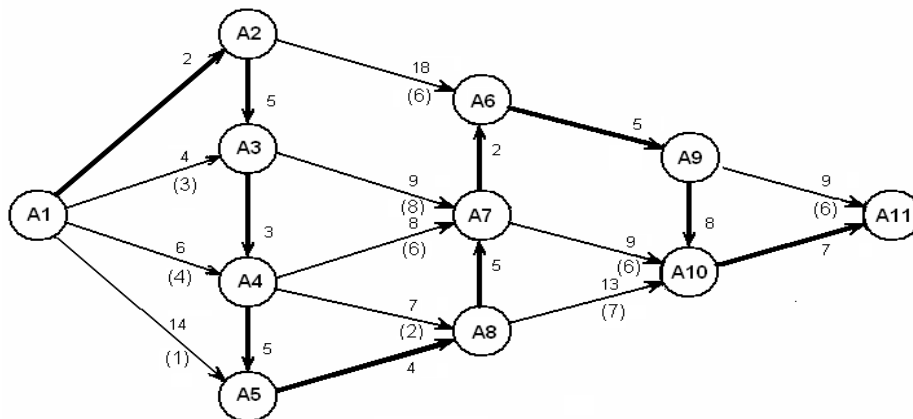

Figure 11. The critical path on the network schedule for a test example of greater dimension

Finally, it is necessary to deals with two important moments confirming an opportunity of practical use of the offered technique.

The first concerns the maximal dimension of a task. As it is well known, the standard tool **Solver** (built in standard **Excel**) has the general limitation on quantity of cells with initial data (the greatest possible to use about 200 cells). For overcoming of this restriction in practical tasks with big dimensions we recommend to use more powerful tool **Premium Solver** (accessible free of charge on a site of the developer), which practically supposes usage of matrixes of any dimensions.

The second moment concerns the uses of newer versions of spreadsheet **Excel**. Though the material stated in the paper has been received in **Excel 2003**, check of a technique in **Excel 2010** shown its working capacity. Certainly, the sequence of commands, their arrangement on panels and names can be others.

3. Conclusions

The described design procedure to find the critical path demands the minimal labour expenditures from the user irrespective of the task's sizes. In spite of the fact that the special methods considering their structure are developed for network models, many network tasks can be solved as a tasks of linear programming (in particular, in transportation).

In this paper the expediency of the solving of the examined tasks by their reduction to problem of the longest path search has been shown. The demanded decision is easy for receiving by tool **Solver** from spreadsheet **Excel** by the offered technique.

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