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# ANALYSIS OF BEHAVIOR OF SOLUTIONS' SUPPORT FOR NONLINEAR PARTIAL EQUATIONS

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***Розглядається початкова задача про розподіл температури на нескінченності для напівлінійного параболічного рівняння другого порядку, що містить член поглинання. Проводиться аналіз поведінки носія розв'язку задачі Коші для зазначеного вище диференціального рівняння в частинних похідних. Доведено, що при певних умовах на параметри задачі спостерігається стиснення носія***

***Ключові слова: розв'язок, задача Коші, диференціальне рівняння в частинних похідних, носій***

***Рассматривается начальная задача о распределении температуры на бесконечности для полулинейного параболического уравнения второго порядка, которое содержит член поглощения. Проводится анализ поведения носителя решения задачи Коши для указанного выше дифференциального уравнения в частных производных. Доказано, что при определенных условиях на параметры задачи наблюдается сжатие носителя***

***Ключевые слова: решение, задача Коши, дифференциальное уравнение в частных производных, носитель***

## 1. Introduction

Partial differential equations of the second order of parabolic type are more common in the study of processes of heat conduction and diffusion. As you know, the process of heat distribution in space can be fully described by temperature u(x, t), where $x \in \mathbb{R}^n$. If the temperature is not constant, then there are heat flows which are directed from places with higher temperature to places with the lowest temperature. We consider thermal processes in a fairly large range of temperature changes, leading to quasi-linear heat equations. So let's write divergent parabolic equations in general form:

$$u_t = \operatorname{div}\left(k\left(u, u_x\right) \nabla u\right) + F\left(x, t\right),$$

where

$$\operatorname{div} A\left(x\right) = \sum_{i=1}^{n} \frac{\partial A_i}{\partial x_i}$$

for

$$A\left(x\right) = \left(A_1\left(x\right), \ldots, A_n\left(x\right)\right);$$

$k\left(u, u_x\right)$ – the coefficient of the thermal diffusivity;

$$\nabla u = \operatorname{grad} u = \frac{\partial u}{\partial x_1 \ldots \partial x_n};$$

$F\left(x, t\right)$ – the density of the heat sources (flows).

An actual and interesting problem is solutions' properties investigation of the initial problem (Cauchy problem) about distribution of temperature at infinity: find a solution of the heat equation

$$u_t = \operatorname{div}\left(k\left(u, u_x\right)\nabla u\right) + F\left(x,t\right)$$

in the domain $x \in \mathbb{R}^n$, $t > 0$, which satisfies the condition:

$$u\left(x,0\right) = u_0\left(x\right),\ x \in \mathbb{R}^n.$$

The issue of existence and uniqueness of the solution for the given problem has been studied by many authors and successfully solved (for instance, in [1, 2]). Moreover, in the case of a particular coefficient of the thermal diffusivity and certain heat flux density, we deal with the process

$$u_t = \sum_{i=1}^{n} \frac{\partial}{\partial x_i}\left(\left|\nabla u\right|^{p-1}\frac{\partial u}{\partial x_i}\right) - \left|u\right|^{\lambda-1}u,\quad x \in \mathbb{R}^n,\ t>0,$$

$$u\left(x,0\right) = u_0\left(x\right),\ x \in \mathbb{R}^n.$$

## 2. Literature review and problem statement

Today it is known [3–6], that for the above-mentioned initial problem, the phenomenon of instantaneous compactification of the carrier of solution holds, when despite the fact that the carrier of the initial function may coincide with the whole space $\mathbb{R}^n$, the support of solution becomes compact in an arbitrarily small time $t>0$ and shrinks in the initial moments. The research is devoted to the study and investigation of this phenomenon.

The paper [7] was the first, where the property of instantaneous shrinking was systematically investigated for the semilinear heat equation

$$u_t = \Delta u + b\left(u\right),\ b\left(0\right) = 0,\ b\left(s\right) > 0\quad \forall s > 0.$$

In [7] the conditions on the behavior of the function b(u) in the neighborhood of zero that guarantee the property of instantaneous compactification for nonnegative, continuous, bounded initial function that tends to zero at infinity have been found.

For variational inequalities, the property of instantaneous shrinking was investigated in [8]. In the papers [9, 10] for the one-dimensional equation

$$u_t = \left(u^m\right)_{xx} - g\left(x\right)u^p$$

the method, based on the comparison principle was applied. It was found, for instance, that if

$$u_0 \le c_0\left(1+\left|x\right|\right)^{-\gamma},\ g\left(x\right) \ge c_1\left(1+\left|x\right|\right)^{-\beta},$$
$$m \ge 1,\ p \in \left(0,1\right),\ \beta > 0,\ \gamma > 0,\ c_i > 0,$$

then the given problem has the property of instantaneous shrinking.

We have to note that a similar phenomenon may occur in other important physical models. So, in [11] for the equation

$$u_t = \left(u^m\right)_{xx} + \left(u^n\right)_x,\ 0 < n < 1,\quad m \ge 1$$

the following result was proved:

$$\text{if } u_0\left(x\right) \sim cx^{-\frac{1}{1-n}}\ \text{ as } x \to \infty, \text{ then}$$

$$u\left(x,t\right) > 0,\ t \in \left(0, \frac{1}{n}c^{1-n}\right),\ x \ge x_0 > 0$$

and the solution u(x, t) has compact support for

$$t > \frac{1}{n}c^{1-n}.$$

Hence, the effect of instantaneous shrinking holds under condition:

$$u_0 = o\left(x^{-\frac{1}{1-n}}\right).$$

Similar results were obtained in [12]. For the second-order quasi-linear parabolic equations of the divergent type with the initial data from $L_q$ see [13].

In [14] the class of parabolic diffusion equations with inhomogeneous source were considered. Two classes are highlighted, the radius of the carrier of the solution depends and does not depend on the geometry of the domain.

But, note here, that the majority of the above-mentioned results have been obtained for non-negative solutions and with the assumption on the initial function: either $u_0 \to 0,\ \left|x\right| \to \infty$, or it has a majorant. The main tool in getting the results was the maximum principle. It turns out that if the initial function has no monotone majorant for example as in the case

$$u_0\left(+k\right) = 1,\ k \in Z,\ u_0\left(x\right) > 0,\quad X \in \mathbb{R},$$

then even for the simplest equation

$$u_t = u_{xx} - u^p,\ 0 < p < 1$$

there are no results, as the comparison principle here is inadequate. For higher order equations, we have no such principles. Hence, there is an actual problem – to find a new approach that will enable to analyze the behavior of the solution in more complex and general situations, which do not impose additional conditions on the function from the Cauchy condition.

So, let us consider the problem:

$$u_t - \sum_{i=1}^{n} \frac{\partial}{\partial x_i}\left(\left|\nabla u\right|^{p-1}\frac{\partial u}{\partial x_i}\right) + \left|u\right|^{\lambda-1}u = 0,\quad x \in \mathbb{R}^n,\ t>0, \qquad (1)$$

$$u(x,0) = u_0(x),\ x \in \mathbb{R}^n, \qquad (2)$$

where $\nabla u$ denotes, as is customary in the literature, the gradient, i. e.:

$$\nabla u = \operatorname{grad} u = \frac{\partial u}{\partial x_1 \ldots \partial x_n};$$

where p and λ – positive real numbers;

the initial function from the Cauchy condition (2) such that $u0(x) \in L_2(\mathbb{R}^n)$; n – space dimensions, $n \ge 1$.

## 3. The purpose and objectives of research

The goal of the work is to study the solutions' behavior for a wide class of nonlinear partial differential equations through the use of a new approach that has been proposed in [3]. Specifically, we are interested in a phenomenon called "compactification" (or "instantaneous shrinking") of the solutions' support supp u(x, t), where

$$\operatorname{supp} u(x,t)=\operatorname{clos}\left\{x\in R^n : u(x,t)\neq 0\right\}.$$

The mathematical formulation of the problem of the given work: to prove that the Cauchy problem (2) for parabolic equations (1) has the shrinking property of support of the solutions. This is an important problem in terms of applied mathematics and mathematical physics.

In order to achieve this goal the following tasks were solved:

– to get integral estimates linking different norms of solution;

– to reduce integral relationships to non-differential inequality and to analyze this inequality;

– to establish the property of shrinking of the support.

## 4. The method of solving the problem

The method of investigation is the result of evolution of ideas coming from the theory of linear parabolic and elliptic equations. It can be applied for different purposes and different equations. The essence of this approach consists in getting and analyzing special (non-differential) inequality linking different energy norms of the solution.

## 5. The result of an investigation of behavior of the carrier of solution for the equation (1)

First of all, we introduce here a definition, it will enable to present the obtained result on a strict mathematical level.

***Definition***. The Cauchy problem (1), (2) has the instantaneous compactification property, if for any t>0 the support of the solution u(x, t) is bounded even if it is unbounded for t=0.

The main result of the research is the following theorem.

***Theorem***. In both of the cases:

– $0<\lambda<1, \quad p\geq 1;$

– $0<\lambda<p,$

– if $\frac{n-2}{n+2}<p<1,$ when $n>2;$

– if $0<p<1,$ when $n\leq 2,$

the problem (1), (2) has the "instantaneous compactification" property.

## 6. The proof of compactification property of the carrier of solution

In order to prove the Theorem about compactification of solutions' support of the problem (1), (2) we need the well-known Gagliardo-Nirenberg interpolation inequality, which will be given below, and the following lemma that is not a trivial fact and, therefore, requires a strict mathematical proof.

***Lemma 1.*** If f(τ, s) is positive, increasing function, which satisfies the inequality

$$f\left(\tau+f^{\alpha}(\tau, s),\ s+f^{\beta}(\tau, s)\right)\leq \delta f(\tau, s); \tag{3}$$

– for each $\tau>\tau_0,\ s>s_0,\ \delta>1,\ \alpha>0,\ \beta>0,$ then:

$$f(\tau, s)\equiv 0;$$

– for all (τ, s) such that:

$$\tau>\tau_0+\frac{1}{1-\delta^{\alpha}}f^{\alpha}(\tau_0,s_0),\quad s>s_0+\frac{1}{1-\delta^{\beta}}f^{\beta}(\tau_0,s_0).$$

*Proof of Lemma 1.*

Define the sequences as follows:

$$\tau_{i+1}=\tau_i+f^{\alpha}(\tau_i,s_i),\ s_{i+1}=s_i+f^{\beta}(\tau_i,s_i),\ i=1,2,\ldots$$

Then from (3) we have

$$f(\tau_{i+1},s_{i+1})\leq \delta f(\tau_i,s_i).$$

After iteration, we obtain

$$f(\tau_{j+1},s_{j+1})\leq \delta^j f(\tau_0,s_0)$$

for each $j\in N.$ Then:

$$\tau_{j+1}=\tau_j+f^{\alpha}(\tau_j,s_j)=\tau_{j-1}+f^{\alpha}(\tau_{j-1},s_{j-1})=f^{\alpha}(\tau_j,s_j)=$$
$$=\tau_0+\sum_{i=0}^{j}f^{\alpha}(\tau_i,s_i)\leq \tau_0+f^{\alpha}(\tau_0,s_0)\times$$
$$\times\sum_{i=0}^{j}\delta^{i\alpha}\leq \tau_0+f^{\alpha}(\tau_0,s_0)\cdot\frac{1}{1-\delta^{\alpha}}.$$

Similarly, it is possible to obtain the inequality:

$$s_{j+1}\leq s_0+f^{\beta}(\tau_0,s_0)\cdot\frac{1}{1-\delta^{\beta}}.$$

From the fact that

$$\lim f(\tau_j, s_j)=0,\ j\to\infty$$

and as the sequences are uniformly bounded, the necessary result follows. Thus, the lemma is proved.

*Proof of Theorem.*

For any numbers

$$0\leq\tau_1<\tau_2\leq T,\quad 0<s_1<s_2<\infty,$$

we define by

$\Omega(s_1)=\left\{x\in R^n : |x|>s_1\right\}$ – exterior of sphere;

$G_{\tau_1}^{\tau_2}(s_1)=\Omega(s_1)\times(\tau_1,\tau_2)$ – exterior of cylinder;

$$K_{\tau_1}^{\tau_2}(s_1,\ s_2-s_1)=G_{\tau_1}^{\tau_2}(s_1)\setminus G_{\tau_1}^{\tau_2}(s_2).$$

Now let us fix

$$\tau>0,\ s>0,\ \Delta\tau>0,\ \Delta s>0$$

and introduce $\eta(x,t)$ and $\eta_1(x)$:

$$\eta=1 \text{ in } G^T_{\tau+\Delta\tau}(s+\Delta s);\quad \eta=0 \text{ in } R^n\times(0,T)\setminus G^T_\tau(s),$$

$$\eta_1=1 \text{ in } \Omega(s+\Delta s),\quad \eta_1=0 \text{ in } \mathbb{R}^n\setminus\Omega(s).$$

Suppose that

$$0\le\eta_k\le\frac{c}{\Delta s},\ \left|\eta_{x_i}\right|\le\frac{c}{\Delta s},\ \left|\eta_{1x_i}\right|\le\frac{c}{\Delta s}.$$

Here $\eta_k=0$ if $\tau+\Delta\tau<t<T$ and $\nabla\eta=0$ if $|x|>s+\Delta s$.

***Definition.*** An energy solution of (1), (2) is the function such that

$$u(x,t)\in C\big((0,T);L_2(R^n)\big)\cap$$
$$\cap L_{1+p}\big((0,T);W^1_{p+1}(R^n)\big)\cap L_{\lambda+1}\big(R^n\times(0,T)\big)$$

and satisfies the integral identity:

$$\int\limits_{R^n}u(x,T_0)v(x,T_0)dx-\int\limits_0^{T_0}\int\limits_{R^n}u(x,t)v_t(x,t)dxdt+$$
$$+\int\limits_0^{T_0}\int\limits_{R^n}\Big[|\nabla u|^{p-1}u_{x_i}v_{x_i}+|u|^{\lambda-1}uv\Big]dxdt=\int\limits_{R^n}u_0(x)v(x,0)dx,$$

where

$$v\in L_{\lambda+1}\big(\mathbb{R}^n\times(0,T)\big)\cap W^{1,1}_{p+2,2}\big(\mathbb{R}^n\times(0,T)\big).$$

Note here, that the existence of solutions in the above sense is well known if $1\le p$ and $0<\lambda\le p$ – see [2–4].

Let

$$E_T(\tau,s)=\int\limits_{G^T_\tau(s)}u^2dxdt,\ \ I_T(\tau,s)=\int\limits_{G^T_\tau(s)}|u|^{p+1}dxdt.$$

If we show that for $\forall\tau>0\ \exists s(\tau)<\infty$:

$$H=H_T(\tau,s):=E_T(\tau,s)+I_T(\tau,s)=0,$$

then (thanks to Lemma 1) we will obtain the Theorem.

Thus, it is enough to show:

$$H_T(0,s)\to0,\ s\to\infty,$$

$$H\big(\tau+H^\alpha,s+H^\beta\big)\le\mu H,\ \alpha>0,\ \beta>0,\ \ 0<\mu<1.$$

Let us substitute $v=u\eta^{p+1}$ into the integral identity and integrate by parts

$$2^{-1}\int\limits_{R^n}u^2(x,T)\eta^{p+1}(x,T)dx+$$
$$+\int\limits_0^T\int\limits_{R^n}\Big[|\nabla u|^{p+1}+|u|^{\lambda+1}\Big]\eta^{p+1}dxdt=$$
$$=(p+1)\int\limits_0^T\int\limits_R(2^{-1}u^2\eta_t+|\nabla u|^{p-1}u_{x_i}u\eta_{x_i})\eta^pdxdt. \tag{4}$$

For the right-hand side of (4) we apply the Young's inequality with ε:

$$\int\limits_{\Omega(s)}u^2\eta^{p+1}dx+\int\limits_{G^T_\tau(s)}\Big(|\nabla u|^{p+1}+|u|^{\lambda+1}\Big)\eta^{p+1}dxdt\le$$
$$\le c\big[I_T+E_T\big]:=cR_1. \tag{5}$$

Let us apply the Gagliardo-Nirenberg inequality:

$$\|v\|_{\alpha,\Omega(s)}\le d_1\|\nabla v\|^\Theta_{\beta,\Omega(s)}\|v\|^{1-\Theta}_\gamma,$$

which used standard notations of norm and indicators

$$\|v\|_{\alpha,\Omega}:=\left(\int\limits_\Omega|v|^\alpha dx\right)^{\frac{1}{\alpha}},\ \frac{1}{\alpha}=\Theta\left(\frac{1}{\beta}-\frac{1}{n}\right)+(1-\Theta)\frac{1}{\gamma},\ \gamma>1,\ \beta>1$$

under $\alpha=2$, $\beta=p+1$, $\gamma=\lambda+1$ and involve the Young's inequality:

$$\left(\int\limits_{\Omega(\bar s)}u^2dx\right)^{1-\nu}\le c\int\limits_{\Omega(\bar s)}\Big(|\nabla u|^{p+1}+|u|\lambda+1\Big)dx,\ \bar s>s_0>0,$$

where

$$\nu=\frac{(p+1)(1-\lambda)}{2(p+1)+n(p-\lambda)}<1.$$

Integration leads to the inequality:

$$\Psi^T_{\bar\tau,\bar s}(1-\nu):=\int_{\bar\tau}^T\left(\int\limits_{\Omega(\bar s)}u^2dx\right)^{1-\nu}dt\le c\int\limits_{G^T_{\bar\tau}(\bar s)}\Big(|\nabla u|^{p+1}+|u|^{\lambda+1}\Big)dxdt.$$

We return back to the integral identity with the test function

$$v=u\eta^{p+1}\chi_l(t),\ l>0,\ \chi_l(t)=\int\limits_0^t\left(\int\limits_{\Omega(s)}u^2\eta^{p+1}dx\right)^l dt,\ t>0$$

and obtain

$$\chi_{l+1}(T)=$$
$$=\chi_l(T)\int\limits_{\Omega(s)}u^2\eta^{p+1}dx+\int\limits_{G^T_\tau(s)}\Big[2|u|^{\lambda+1}\eta^{p+1}-u^2\big(\eta^{p+1}\big)_t\Big]\chi_l(t)dxdt+$$
$$+\int\limits_{G^T_\tau(s)}\Big[2|\nabla u|^{p-1}u_{x_i}\big(u\eta^{p+1}\big)_{x_i}\Big]\chi_l(t)dxdt,$$

from which and (5) we have:

$$\chi_{l+1}(T)\le c\chi_l(T)R_1.$$

By the Holder inequality and integrating the latter inequality:

$$\chi_l(T)\le c\chi_\delta(T)R_1^{l-\delta}\ \text{ for some } l>\delta>0.$$

According to the definition of η(x, t) and previous computations, we obtain several inequalities, which are crucial:

$$\Psi^T_{\tau+\Delta\tau,s+\Delta s}(l)\le\chi_l(T)\le\Psi^T_{\tau,s}(l),$$

$$\Psi^T_{\tau,s}(1-\nu)\le cR_1(s,\Delta s,\tau,\Delta\tau), \tag{6}$$

$$\chi_1(T) \le c\chi_{1-\nu}(T) R_1^{\nu}(s, \Delta s, \tau, \Delta\tau), \quad (7)$$

$$\chi_{1-\nu}(T) \le \Psi^T_{\tau,s}(1-\nu). \quad (8)$$

Now by the definition of the energy function $E_T$:

$$\Psi^T_{\tau+\Delta\tau, s+\Delta s}(1) := E_T(\tau+\Delta\tau, s+\Delta s) \le \chi_1(T). \quad (9)$$

Substitute (7) into (9) and using (8) and (6) we obtain that

$$E_T(\tau+\Delta\tau, s+\Delta s) \le cR_1^{1+\nu}(s, \Delta s, \tau, \Delta\tau). \quad (10)$$

Note here, starting from this point we should distinguish three possible cases:

p=1;
p>1;
0<p<1,

as further (and final) proof course depends precisely on the value of the parameter p.

***Case p=1.***
If p=1, then

$$I_T(\tau, s) = E_T(\tau, s).$$

So, the proof is trivial, because it immediately follows that:

$$\forall \tau > 0\, \exists s(\tau) < \infty : H = H_T(\tau, s) :=$$
$$:= E_T(\tau, s) + I_T(\tau, s) = 2 \cdot E_T(\tau, s),$$

Thus, by (10) and thanks to Lemma 1 we have the result of Theorem.

**Case p>1.**
Now let us a consider nontrivial case, when the parameter is greater than 1, i. e. p>1. Put in the integral identity

$$\alpha = p+1,\ \beta = p+1,\ \gamma = 2.$$

After integrating in t, using the Holder inequality

$$I_T(\tau+\Delta\tau, s+\Delta s) \le$$
$$\le c\left(\int_{G^T_{\tau+\Delta\tau}(s+\Delta s)} |\nabla u|^{p+1} dxdt\right)^{\theta_1} \left(\Psi^T_{\tau+\Delta\tau, s+\Delta s}\left(\frac{p+1}{2}\right)\right)^{1-\theta_1}, \quad (11)$$

where

$$\theta_1 = \frac{n(p-1)}{2(p+1)+n(p-1)} < 1.$$

Inequalities (6)–(8) under

$$l = \frac{1+p}{2} \text{ and } \delta = 1-\nu$$

lead to the following correlation

$$\Psi^T_{\tau+\Delta\tau, s+\Delta s}\left(\frac{p+1}{2}\right) \le c\Psi^T_{\tau,s}(1-\nu) R_1^{\frac{1+p}{2-1+\nu}}.$$

Using the result of (10) to the last estimate we obtain

$$\Psi^T_{\tau+\Delta\tau, s+\Delta s}\left(\frac{p+1}{2}\right) \le cR_1^{\frac{1+p}{2+\nu}}.$$

If we apply the latter inequality to the ratio (11), then

$$I_T(\tau+\Delta\tau, s+\Delta s) \le cR_1^{1+\nu_1},$$
$$\nu_1 = (1-\theta_1)\left(\frac{p-1}{2}+\nu\right) = \frac{\nu(p-\lambda)}{1-\lambda} > \nu. \quad (12)$$

Add (10) and (12), use the definition of the function $R_1$,

$$H_T(\tau+\Delta\tau, s+\Delta s) \le$$
$$\le c_0 \Delta_\tau E_T(\tau, s)\left[\frac{(\Delta_\tau E_T(\tau,s))^{\nu}}{(\Delta\tau)^{1+\nu}} + \frac{(\Delta_\tau E_T(\tau,s))^{\nu_1}}{(\Delta\tau)^{1+\nu_1}}\right] +$$
$$+ c_0 \Delta_s I_T(\tau, s)\left[\frac{(\Delta_s I_T(\tau,s))^{\nu}}{(\Delta s)^{(1+p)(1+\nu)}} + \frac{(\Delta_s I_T(\tau,s))^{\nu_1}}{(\Delta s)^{(1+p)(1+\nu_1)}}\right],$$

where

$$\Delta_\tau f(\tau, s) := f(\tau, s) - f(\tau+\Delta\tau, s),$$

$$\Delta_s f(t, s) := f(\tau, s) - f(\tau, s+\Delta s).$$

Now let us fix

$$\Delta s = (I_T(\tau, s))^{\frac{\nu}{(p+1)(\nu+1)}},\ \Delta\tau = (E_T(\tau, s))^{\frac{\nu}{1+\nu}}.$$

As E and I are monotone, we come to the inequality

$$H_T\left(\tau + H_T^{\frac{\nu}{1+\nu}}(\tau, s,), s + H_T^{\frac{\nu}{(1+p)(1+\nu)}}(\tau, s)\right) \le \mu_1 H_T(\tau, s). \quad (13)$$

In ***case 0<p<1*** it is easy (using the same approach) to obtain an inequality similar to (13), which is to complete a series of computations of our proof, but, of course, with another index, namely,

$$\nu_1 = \frac{\nu(p-\lambda)}{1-\lambda} < \nu.$$

## 7. The discussion of the result about the behavior of the carrier of solution

The results, for example, [7, 8, 11] have been obtained for non-negative solutions and with the assumption on the initial function: either this function tends to zero when $|x| \to \infty$, or has a majorant. To sum up, note here, that if the initial function does not have a monotone majorant, then even for the simplest equation such as

$$u_t = u_{xx} - u^p,\ 0 < p < 1$$

we cannot give an answer about the behavior of the solution. This fact prompted the author to continue earlier research. Furthermore, we emphasize here that the authors [5, 10] and others applied the maximum principle for investigations.

But, unfortunately, for equations of higher order, we have no such principles. Thus, an actual problem arises, which was solved in this work – to adapt a more universal approach to the study of (1), (2), which allows to analyze the behavior of the solution in more complex and common situations.

## 8. Conclusions

As a result of the research:

– relationships, which contain $L_2$, $L_{q+1}$ and $L_{\lambda+1}$ norms of solution were found;

– the functional dependence of the kind of (3) by applying the Young's, Holder, Gagliardo-Nirenberg inequalities to integral estimates was obtained; analysis of the ratios (3), (10), (13) was done;

– it was proved that a carrier of the solution of the problem (1), (2) is bounded for t>0.

Note here that an interesting and important (but separate) problem that has not been realized in this investigation is to estimate the size of the support. On the issue of finding the estimates of a carrier of solution, see the works [15–17].

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