

**MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE**  
**SIMON KUZNETS KHARKIV NATIONAL UNIVERSITY OF ECONOMICS**

## **ECONOMIC AND MATHEMATICAL METHODS**

**Syllabus**  
**for Bachelor's (first) degree**  
**students of speciality**  
**292 "International Economic Relations"**

**Kharkiv**  
**S. Kuznets KhNUE**  
**2018**

UDC 330.45(07.034)

E42

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Затверджено на засіданні кафедри вищої математики й економіко-математичних методів.

Протокол № 1 від 28.08.2017 р.

*Самостійне електронне текстове мережеве видання*

**Economic and Mathematical Methods** : syllabus for Bachelor's E42 (first) degree students of speciality 292 "International Economic Relations" [Electronic resource] / compiled by L. Malyarets, Ie. Misiura. – Kharkiv : S. Kuznets KhNUE, 2018. – 87 p. (English)

The thematic plan of the academic discipline and its content are given according to the modules and themes. Plans of lectures and practical trainings, material for students' knowledge consolidation (test questions, tasks for independent work) as well as methods of students' knowledge assessment according to the credit transfer system of studies are presented.

For Bachelor's (first) degree students of speciality 292 "International Economic Relations".

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## Introduction

The task of the economic and mathematical modelling is construction of models of economic objects and processes in order to describe, optimize, analyze, forecast, provide analytical support for these objects and processes when making decisions at all levels of management. Thus, modelling is a fundamental basis of the methodology of the economy management. Economic and mathematical models are constructed with the help of mathematical methods. Economic and mathematical methods are tools for learning and investigation of economic systems of different complexity. They form a fundamental basis for solving real analytical problems in different fields of activity of management subjects.

The fundamental basis of the mathematical training of economists and managers is the academic discipline "Economic and Mathematical Methods" which is a compulsory discipline of the natural scientific series and a component of the structural logical scheme which is provided for the educational professional program of Bachelor's (first) degree students of specialization 292 "International Economic Relations".

The basic problems of teaching the academic discipline are: giving students knowledge of the basic parts of economic and mathematical methods; raising the level of the fundamental mathematical training of students with intensification of its applied direction; mastering the fundamentals of economic and mathematical methods and application of this knowledge to the economic investigations for solving economic problems; forming skills in the use of the elements of economic and mathematical methods in investigations where mathematical methods (mathematical programming and econometrics) are applied as an instrument of investigation and solving optimization economic problems for forming models of economic processes and developments, acquiring the necessary theoretical and practical knowledge for solving specific problems of economic and mathematical modelling, and obtaining the required mathematical knowledge for the study of other disciplines.

The basic condition for mastering of this discipline is students' knowledge of Higher Mathematics, Probability Theory and Mathematical Statistics, and Economic Theory, Microeconomics, Manufacturing Organization.

# 1. Description of the academic discipline

Name of indicators	Subject area, speciality, academic level	Academic discipline features	
		day-time form of study	distant form of study
Number of credits: 5 for the full-time form; 6 for the distant form	Subject area 29 "International relations"	Compulsory	
Number of thematic modules: 2	Specialization 292 "International Economic Relations"	Academic year	
Total number of hours: 150 for the day-time form; 180 for the distant form		1st	1st
		Term	
The number of hours for the day-time form of studies per week: in class: 4; student's independent work: 5		2nd	1st
		Lectures	
		28 hours	16 hours
	Practical studies		
16 hours		16 hours	
Laboratory studies		16 hours	–
Independent work		90 hours	148 hours
Form of control		Exam	

*Note.* The ratio of the number of class hours and independent work is:  
67 % for the full-time form of studies;  
22 % for the distant form of studies.

## 2. The main purpose and tasks of the academic discipline

**The main purpose** of teaching is to form future specialists' basic mathematical knowledge for solving theoretical and practical problems in professional activity of a competent specialist in any sphere of their activity, skills in analytical thinking and skills in using mathematical knowledge for formation of real processes and developments, and for solving economic problems.

**The main tasks** that should be carried out in the process of teaching the discipline are: giving students knowledge of the basic parts of economic and mathematical methods; definitions, theorems, rules; proving the main

theorems; mastering the fundamentals of the methodology of mathematical investigation of the applied economic problems; independent broadening of knowledge, development of logical and algorithmical thinking; obtaining primary skills in independent learning of mathematical and applied literary sources by students.

**The subject** of the academic discipline "Economic and Mathematical Methods" is the fundamentals of mathematical programming and econometrics.

In the process of learning the academic discipline "Economic and Mathematical Methods" a student receives analytic and investigatory competences which are necessary for modern economists in any sphere of their activity.

The syllabus of the academic discipline "Economic and Mathematical Methods" is compiled according to the statements of the field standard of the higher education of the Ministry of Education and Science of Ukraine based on the educational professional program of bachelor training, which is made by the Scientific Methodical Committee of Economics and Enterprise of the Ministry of Education and Science of Ukraine.

A student starts studying the academic discipline "Economic and Mathematical Methods" in the second term of the first year of studies.

In the process of learning, students obtain the required theoretical knowledge during lectures and acquire practical skills at the practical and laboratory studies and during independent work and fulfillment of individual tasks. Independent and individual work of students is of great value in the process of mastering the material and consolidating knowledge. All of these types of studies were devised according to the statements of the Bologna Declaration.

As a result of studying the academic discipline a student **must know:**

basic mathematical optimization methods as an instrument of construction of optimization models for solving problems in economics;

basic mathematical econometric methods as an instrument of construction of econometric models for solving problems in economics;

basic principles of construction of economic and mathematical models for investigation of optimization problems;

possibilities and restrictions of mathematical methods which are used for solving optimization problems in economics;

basic principles of construction of multidimensional econometric models which are used for solving problems of quantitative analysis and forecasting processes of functioning and development of the economic system;

ways of evaluation of accuracy and reliability of methods which are used for processing of empirical data;

***be able to:***

fulfill statement and formalization of practical tasks according to the general technology of modelling in economics;

classify models of mathematical programming problems and choose the mathematical methods for solving them;

solve economic problems with the help of methods of linear programming;

construct a model of a dual problem, define a solution to the initial problem according to the solution to a dual problem and give an economic explanation of dual estimations;

investigate constancy of an optimal plan relative to the influence of coefficients of an objective function and the right part of the basic system of constraints;

solve transportation problems with the help of the method of potentials;

solve separate problems of nonlinear programming using the graphical method and the method of Lagrange's multipliers;

use the elements of game theory for solving pair matrix games with a zero sum;

construct a pair linear regression equation;

carry out the choice of factors for construction of a multiple regression equation;

define statistical estimations of parameters of a multifactorial econometric model and verify their statistic significance;

carry out estimation of parameters of a regression equation for time series and define the ability of this equation to forecast;

use statistical tests for defining the correspondence of an econometric model which is constructed with the help of sample data to basic hypothesis which lies in its basis;

construct economic models on the basis of the system of structural equations;

construct dynamic econometric models;

model one-dimensional time series;

analyse time series;

learn mathematical literature by oneself;

broaden the knowledge, develop logical and algorithmic thinking by oneself;

use mathematical methods as an instrument of the economic and mathematical modelling for solving practical economic problems.

A modern tendency in higher education is the reorientation of students of higher educational institutions from the process of education to the result, from knowledge to skills, forming definite competences.

In the process of learning the academic discipline "Economic and Mathematical Methods" a student receives analytic and investigatory competences which are required for a modern economist in any sphere of their activity (Table 2.1).

Table 2.1

**Competences which are formed as a result of mastering  
the academic discipline "Economic and Mathematical Methods"**

The code of the competence	The name of the competence	The components of the competence
1	2	3
AOM	Forming analytic thinking. Development of the ability to construct mathematical and economic models. Forming the ability to use basic mathematical optimization methods as an instrument of construction of optimization models when solving practical problems in economics	A student must 1) solve problems of linear programming; 2) form a dual problem for the given primal problem; 3) find a solution to a dual problem knowing a solution to a primal problem with the help of duality theorems; 4) give economic interpretations of solutions to primal and dual problems; 5) be able to solve and find an optimal solution to transportation problems; 6) be able to solve problems of integer programming; 7) solve problems of nonlinear programming; 8) solve problems of game theory

Table 2.1 (the end)

1	2	3
AEM	<p>Forming analytic thinking and skills in independent formation of economic and mathematical models for description of different economic processes.</p> <p>Forming the ability to use basic mathematical econometric methods as an instrument of construction of econometric models when solving practical problems in economics</p>	<p>A student must</p> <ol style="list-style-type: none"> <li>1) construct a pair regression equation;</li> <li>2) check the quality of the constructed pair linear model;</li> <li>3) verify the statistical significance of coefficients of regression and correlation;</li> <li>4) check the adequacy of econometric models;</li> <li>5) construct a multiple regression equation;</li> <li>6) check the statistical significance of coefficients of a regression equation;</li> <li>7) use methods of elimination of multicollinearity;</li> <li>8) use methods of defining heteroscedasticity;</li> <li>9) use methods of elimination of autocorrelation;</li> <li>10) use advanced methods of regression analysis;</li> <li>11) use systems of econometric equations;</li> <li>12) use methods of partitioning of dynamic econometric models</li> </ol>

\* Application of optimization methods (AOM).

\* Application of econometric methods (AEM).

The structure of professional competences and forming them according to the National Scale of Qualifications of Ukraine is given in Appendix A.

### 3. The themes of the academic discipline

#### Thematic module 1 Optimization methods

##### Theme 1. The general theoretical foundations of optimization methods and models in economics

###### 1.1. Construction of economic and mathematical models.

A conceptual statement of the problem of construction of economic and mathematical models. The place of modelling among the methods of learning social and economic systems. The objective of modelling. Defining models, properties of models.



*1.2. The choice of a method for solving economic and mathematical problems.*

The choice of a method for solving economic and mathematical problems and explanation of this choice. Possibilities of realization of search for solution to optimization problems with the help of the package of applied programs for personal computer.

*1.3. Foundations of the classical optimization theory.*

Foundations of the classical optimization theory. General remarks. Classification of problems. Information support for economic and mathematical optimization models.

*1.4. Optimization and mathematical programming problems.*

The statement of an optimization problem. The conditional extremum. Lagrange's method of multipliers. The economic meaning of Lagrange's multipliers. The iterative method of solving problems of mathematical programming.

## **Theme 2. Problems of linear programming and methods for solving them**

*2.1. Statements and basic definitions of linear programming problems.*

Economic and mathematical statements of linear programming problems (LPP). The system of hypothesis. Basic definitions. A standard form of a linear optimization model. A set of feasible solutions and an optimal solution to LPP.

*2.2. The graphical method of solving LPP.*

The graphical method of solving LPP. The geometrical meaning of LPP. The graphical method of solving LPP, its possibilities and the field of application. Examples of problems, which can be solved by the graphical method.

*2.3. The simplex method of solving LPP.*

The simplex method of solving LPP. The canonical (basic) form of LPP. The construction (plotting) of support solutions. The optimization criterion. Searching for an optimal solution using the algorithm of the simplex method. The geometrical meaning of the simplex method. The theoretical aspects of the simplex method. A problem with mixed constraints.

*2.4. The method of artificial basis.*

The method of artificial basis. The features of solving LPP which are given in the general form of LPP for solving economic problems.

### **Theme 3. Duality theory and analysis of linear models of economic optimization problems**

#### *3.1. Basic definitions, problems and theorems of the duality theory.*

The basic concepts of duality theory. Mutual dual problems of linear programming. The economic meaning of the primal and dual LPP as an example of the product mix problem. Rules of construction of a mathematical model of the dual problem.

#### *3.2. The theorems of the duality theory.*

The basic duality theorems and their economic explanation. Finding an optimal solution to the initial problem using the solution to the dual problem.

#### *3.3. Postoptimization analysis of LPP.*

Postoptimization analysis of LPP. Dual estimations and the shortage of resources in the neighbourhood of the optimal solution to LPP.

#### *3.4. The analysis of a range of coefficients of basic components.*

The analysis of a range of changes of components of the matrix-column of the right part of the basic system of constraints. The analysis of a range of changes of coefficients of the objective function. The analysis of a range of changes of coefficients of the basic matrix of the constraints system.

### **Theme 4. The transportation problem**

*4.1. The basic statement of the transportation problem and methods of finding the support basic solution.*

Solving the transportation problem using the criterion of costs. The statement of the transportation problem using the criterion of the transportation cost. Finding the support basic solution. Transformation of the other basic solution. The problem of solution degeneracy of the transportation problem and ways to eliminate the degeneracy.

#### *4.2. The method of potentials.*

Finding an optimal solution using the method of potentials. The optimality criterion of the solution. The method of potentials. The economic meaning of potentials.

#### *4.3. Investigation of stability of an optimal solution.*

Investigation of stability of an optimal solution as a problem of parametric programming. Transportation problems with additional conditions.

#### *4.4. Solving a transportation problem using the criterion of time.*

Solving a transportation problem using the criterion of time. Problems with economic content which are reduced to transportation problems.

## **Theme 5. Integer programming**

### *5.1. The basic statement of integer programming problems.*

The economic statement of the integer programming problem and its mathematical model.

### *5.2. Types of integer programming problems.*

An assignment problem. Solving an assignment problem as a transportation problem. The investment portfolio as a problem of combinatorial optimization.

### *5.3. The geometrical meaning of solutions.*

The geometrical meaning of solutions to the integer programming problem on a plane.

### *5.4. The methods of solving integer programming problems.*

The general characteristic of the methods of solving integer problems: cutting methods, combinatorial methods, methods of approximate computations. The branch and bound method. Gomory method (the cutting method). Generation of additional constraints.

### *5.3. Examples of integer programming problems.*

Examples of economic problems which require using models of integer programming.

## **Theme 6. Nonlinear optimization models of economic systems**

### *6.1. The statement and the geometrical meaning of a nonlinear programming problem.*

Economic and mathematical statements of a problem of nonlinear programming. The geometrical meaning of a nonlinear programming problem.

### *6.2. Basic difficulties of solving nonlinear programming problems*

Basic difficulties which arise in solving nonlinear programming problems.

### *6.3. Necessary and sufficient conditions and theorems.*

The necessary and sufficient conditions of the existence of a saddle point. Kuhn – Tucker theorem.

### *6.4. The bases of quadratic programming problems.*

The economic statement and mathematical models of some quadratic programming problems.

### *6.5. The bases and methods of solving convex programming problems.*

Problems of convex programming and methods of solving them.

*6.6. The bases and methods of solving linear fractional programming problems.*

The economic and mathematical statement of a linear fractional programming problem. The geometrical meaning of a linear fractional programming problem. Solving a linear fractional programming problem with the help of reducing it to a linear programming problem.

### **Theme 7. Game theory. Analysis and risk management in economics on the basis of the concept of game theory**

*7.1. The basic definitions and models of game theory.*

The basic notions of game theory, a mathematical model of the matrix game as a particular case of probabilistic models of economic systems.

*7.2. The basic notions and the basic theorem of two players' matrix games.*

Matrix games of two players. The payoff matrix. Minimax and maximin criteria. The game price. The game in pure strategies. A saddle point. The game in mixed strategies. The basic theorem of game theory (Neumann theorem). Reducing a matrix game of two players to a linear programming problem.

*7.3. A graphical method of solving a matrix game.*

The geometrical meaning of the matrix game of two players. Finding active strategies of players. The content of basic ways of quantitative pricing risk. The system of quantitative estimations of the measure of an economic risk. A game price as the risk appraisal.

## **Thematic module 2 Econometric methods**

### **Theme 8. Particular properties of construction of econometric models and ways of construction**

*8.1. Particular properties of construction of econometric models.*

Particular properties of econometric models. The role and place of econometric models in the analysis of socioeconomic systems. An economic model and problems of econometric modelling.

*8.2. Properties of observations and data.*

Forming a set of observations. The concept of homogeneity of observations. The accuracy of initial data.

*8.3. The basic steps of construction of an econometric model.*

The basic steps of construction of an econometric model. The general characteristic of the basic steps of construction of an econometric model.

#### *8.4. Checking the statistical significance and quality of a model.*

Particular properties of interpreting of the form of an econometric model. Checking the statistical significance of a model. Characteristics and criteria of the quality of econometric models. Statistical estimations of parameters of econometric models.

### **Theme 9. A pair linear model**

#### *9.1. The content and the estimator of parameters.*

The specification of a model. Linear regression and correlation: the content and the estimator of parameters.

#### *9.2. The method of least squares.*

Estimation of parameters of a linear model of pair regression with the help of the method of least squares.

#### *9.3. Checking the quality and statistical significance of the constructed pair linear model.*

Checking the quality of the constructed pair linear model. The estimator of the statistical significance of coefficients of regression and correlation. Nonlinear regression.

#### *9.4. Variance analysis.*

Variance analysis. The determination coefficient. Checking the adequacy of an econometric model.

### **Theme 10. Methods of construction of a multiple regression model**

#### *10.1. The basic steps and specification of construction of a multiple regression model.*

The general questions of construction of a multiple regression model. The specification of a model. Estimation of parameters of a regression equation. Methods of construction of general linear regression.

#### *10.2. The method of least squares.*

The method of least squares (MLS), statistical properties of MLS-estimators. The variance and standard errors of parameters of an equation.

#### *10.3. Analysis of the quality of a multiple linear regression equation and interval estimators of its coefficients.*

Estimation of parameters of a linear equation of multiple regression. Interval estimators of coefficients of a theoretical regression equation. Analysis of the quality of an empirical equation of multiple linear regression. Partial equations of a regression.

Multiple and partial correlation.

*10.4. Checking the statistical significance and general quality of a regression equation.*

Checking the statistical significance of coefficients of a regression equation.  
Checking the general quality of a regression equation.

*10.5. Forecasting and accuracy of prediction.*

Forecasting according to regression models. The accuracy of prediction.

### **Theme 11. Problems in the construction of linear multiple regression models**

*11.1. Sequences and methods of elimination of multicollinearity.*

Different aspects of multiple regression and problems which arise as a result of disorder of conditions of using MLS. Multicollinearity and its sequences. Methods of elimination of multicollinearity.

*11.2. Sequences and methods of elimination of heteroscedasticity.*

Heteroscedasticity and methods of defining heteroscedasticity. The generalized method of least squares.

*11.3. Sequences and methods of elimination of autocorrelation.*

Autocorrelation of model residuals and methods of elimination of autocorrelation. Implications of autocorrelation of model residuals. Durbin – Watson test. Methods of elimination of autocorrelation of model residuals.

*11.4. Autoregression*

Estimation of parameters of a model with autoregression.

*11.5. Interpreting the obtained linear multiple regression models.*

Interpreting problems of parameters of a multifactor model.

### **Theme 12. The generalized schemes of regression analysis**

*12.1. The generalized schemes and methods of regression analysis.*

The generalized schemes of regression analysis.

*12.2. The generalized methods of regression analysis.*

The generalized Aitken method of least squares. Dummy variables. Advanced methods of regression analysis.

### **Theme 13. The systems of econometric equations**

*13.1. The basic notions of the systems of econometric equations.*

The general concept of the systems of equations, which are used in econometrics. The structure and reduced forms of a model. An identification problem. Estimation of parameters of a structural model.

### *13.2. The method of solving systems of econometric equations.*

The two-stage method of least squares (2SMLS). Economic models on the basis of the system of structural equations.

## **Theme 14. Dynamic econometric models**

### *14.1. Econometric models with lag variables.*

The concepts of lag and lag variables. The general characteristics of models with separated lags. The types of lag models. The meaning of parameters of models with a separated lag.

### *14.2. The basic methods.*

Defining the lag structure. Lags of independent variables. A mutual correlation function. A correlogram. The choice of a model type with separated lags. Methods of partitioning of dynamic econometric models. Almon's method. Koyck's method.

## **4. The structure of the academic discipline**

From the very beginning of studying the academic discipline each student has the possibility to learn both the discipline syllabus and forms of organization of education, as well as the structure, contents and volume of each of its educational modules, and all types of control and methods of the educational work assessment.

The educational process according to the syllabus of the academic discipline "Economic and Mathematical Methods" is realized in such forms as: lectures, practical and laboratory studies; fulfillment of students' independent work; control activities.

A student's mastering of the academic discipline is carried out with the help of consecutive and thorough learning of the educational modules. An educational module is a relatively separate block of the given discipline, which logically unites its educational elements by content and interconnections. The assessment of knowledge and skills obtained by a student while learning the material of each module is effected in the final module control.

The thematic plan of the academic discipline consists of two thematic modules (Table 4.1).

### The structure of the test credit of the academic discipline

Names of thematic modules and themes	The number of hours											
	the day-time form of studies						the distant form of studies					
	total	which are allocated for					which are allocated for					
		lecture	practical	laboratory	final control	independent work: preparation for studies	total	lecture	practical	laboratory	final control	independent work: preparation for studies
1	2	3	4	5	6	7	8	9	10	11	12	13
<b>Thematic module 1 Optimization methods</b>												
<i>Theme 1.</i> The general theoretical foundations of optimization methods and models in economics	10	2	1	2	–	5	7	1	1	–	–	5
<i>Theme 2.</i> Problems of linear programming and methods for solving them	10	2	1	2	–	5	16	2	2	–	–	12
<i>Theme 3.</i> Duality theory and analysis of linear models of economic optimization problems	8	2	2	–	–	4	12	1	1	–	–	10
<i>Theme 4.</i> The transportation problem	10	2	1	2	–	5	14	1	1	–	–	12
<i>Theme 5.</i> Integer programming	7	2	1	–	–	4	10	1	1	–	–	8
<i>Theme 6.</i> Nonlinear optimization models of economic systems	9	2	1	1	–	5	10	1	1	–	–	8



Table 4.1 (the end)

1	2	3	4	5	6	7	8	9	10	11	12	13
<i>Theme 7. Game theory. Analysis and risk management in economics on the basis of the concept of game theory</i>	10	2	1	1	–	6	14	1	1	–	–	12
<b>Total for module 1</b>	<b>64</b>	<b>14</b>	<b>8</b>	<b>8</b>	<b>–</b>	<b>34</b>	<b>83</b>	<b>8</b>	<b>8</b>	<b>–</b>	<b>–</b>	<b>67</b>
<b>Thematic module 2 Econometric methods</b>												
<i>Theme 8. Particular properties of construction of econometric models and ways of construction</i>	7	2	1	–	–	4	8	1	1	–	–	6
<i>Theme 9. A pair linear model</i>	11	2	1	2	–	6	20	2	2	–	–	16
<i>Theme 10. Methods of construction of a multiple regression model</i>	13	2	2	2	–	7	18	1	1	–	–	16
<i>Theme 11. Problems in the construction of linear multiple regression models</i>	12	2	1	2	–	7	12	1	1	–	–	10
<i>Theme 12. The generalized schemes of regression analysis</i>	12	2	1	2	–	7	12	1	1	–	–	10
<i>Theme 13. The systems of econometric equations</i>	9	2	1	–	–	6	7	1	1	–	–	5
<i>Theme 14. Dynamic econometric models</i>	8	2	1	–	–	5	6	1	1	–	–	4
<b>Total for module 2</b>	<b>72</b>	<b>14</b>	<b>8</b>	<b>8</b>	<b>–</b>	<b>42</b>	<b>83</b>	<b>8</b>	<b>8</b>	<b>–</b>	<b>–</b>	<b>67</b>
<i>Preparation for the exam</i>	10	–	–	–	–	10	10	–	–	–	–	10
<i>Preexam consultations</i>	2	–	–	–	2	–	2	–	–	–	2	–
<i>Exam</i>	2	–	–	–	2	–	2	–	–	–	2	–
<b>Total number of hours</b>	<b>150</b>	<b>28</b>	<b>16</b>	<b>16</b>	<b>90</b>	<b>180</b>	<b>16</b>	<b>16</b>	<b>–</b>	<b>–</b>	<b>–</b>	<b>148</b>

## 5. The plan of practical studies

### 5.1. The themes of practical studies

A **practical study** is a form of educational studies, at which the lecturer organizes a detailed consideration of separate theoretical statements of the academic discipline and forms the abilities and skills in their practical application through the students' individual accomplishment of the formulated tasks.

The plan of the practical studies, their content and a bibliography for each theme are given in Table 5.1.

Table 5.1

**The plan of practical studies**

The name of the thematic module	The themes of the practical studies (by modules)	The number of hours	Recommended reading
1	2	3	4
<b>Thematic module 1. Optimization methods</b>	<p><i>Theme 1. The general theoretical foundations of optimization methods and models in economics.</i></p> <p>1. The objective of modelling. 2. Defining models, properties of models. 3. The choice of a method for solving economic and mathematical problems and explanation of this choice</p>	1	<p>Main: [1 – 4; 6; 9]. Additional: [10; 13]. Methodological support: [15 – 17]</p>
	<p><i>Theme 2. Problems of linear programming and methods for solving them.</i></p> <p>1. Economic and mathematical statements of linear programming problems (LPP). 2. The graphical method of solving LPP. 3. Search of an optimal solution using the algorithm of the simplex method</p>	1	<p>Main: [1 – 4; 6; 9]. Additional: [10; 13]. Methodological support: [15 – 17]</p>
	<p><i>Theme 3. Duality theory and analysis of linear models of economic optimization problems.</i></p> <p>1. Mutual dual problems of linear programming. 2. Rules of construction of a mathematical model of the dual problem. 3. Finding an optimal solution to the initial problem using the solution to the dual problem</p>	2	<p>Main: [1 – 4; 6; 9]. Additional: [10; 13]. Methodological support: [15 – 17]</p>

Table 5.1 (continuation)

1	2	3	4
	<p><i>Theme 4. The transportation problem.</i></p> <ol style="list-style-type: none"> <li>1. The statement of the transportation problem using the criterion of the transportation cost.</li> <li>2. Finding the support basic solution.</li> <li>3. Finding the optimal solution using the method of potentials.</li> <li>4. Solving a transportation problem using the criterion of time</li> </ol>	1	<p>Main: [1 – 4; 6; 9]. Additional: [10; 13]. Methodological support: [15 – 17]</p>
	<p><i>Theme 5. Integer programming.</i></p> <ol style="list-style-type: none"> <li>1. The economic statement of the integer programming problem and its mathematical model.</li> <li>2. The geometrical meaning of solutions to the integer programming problem on a plane.</li> <li>3. The branch and bound method.</li> <li>4. Gomory method (the cutting method)</li> </ol>	1	<p>Main: [1 – 4; 6; 9]. Additional: [10; 13]. Methodological support: [15 – 17]</p>
	<p><i>Theme 6. Nonlinear optimization models of economic systems.</i></p> <ol style="list-style-type: none"> <li>1. Economic and a mathematical statements of a problem of nonlinear programming</li> <li>2. Necessary and sufficient conditions of the existence of a saddle point.</li> <li>3. Problems of convex programming and methods for solving them.</li> <li>4. The economic and mathematical statement of a linear fractional programming problem and its geometrical meaning</li> </ol>	1	<p>Main: [1 – 4; 6; 9]. Additional: [10; 13]. Methodological support: [15 – 17]</p>
	<p><i>Theme 7. Game theory. Analysis and risk management in economics on the basis of the concept of game theory.</i></p> <ol style="list-style-type: none"> <li>1. A game in pure strategies.</li> <li>2. A saddle point. A game in mixed strategies.</li> <li>3. The geometrical meaning of the matrix game of two players. Finding active players' strategies</li> </ol>	1	<p>Main: [1 – 4; 6; 9]. Additional: [10; 13]. Methodological support: [15 – 17]</p>
<p><b>Thematic module 2. Econometric methods</b></p>	<p><i>Theme 8. Particular properties of construction of econometric models and ways of construction.</i></p> <ol style="list-style-type: none"> <li>1. The basic steps of construction of an econometric model.</li> <li>2. Checking the statistical significance of a model</li> </ol>	1	<p>Main: [1 – 9]. Additional: [10; 12; 13]. Methodological support: [15 – 17]</p>

Table 5.1 (continuation)

1	2	3	4
	<p><i>Theme 9. A pair linear model.</i></p> <ol style="list-style-type: none"> <li>1. Linear regression and correlation.</li> <li>2. Estimation of parameters of a linear model of pair regression with the help of the method of least squares.</li> <li>3. The statistical significance of coefficients of regression and correlation</li> </ol>	1	<p>Main: [1 – 9]. Additional: [10; 12; 13]. Methodological support: [15 – 17]</p>
	<p><i>Theme 10. Methods of construction of a multiple regression model.</i></p> <ol style="list-style-type: none"> <li>1. Methods of construction of general linear regression.</li> <li>2. The method of least squares (MLS).</li> <li>3. The variance and standard errors of parameters of an equation.</li> <li>4. Analysis of the quality of an empirical equation of multiple linear regression</li> </ol>	2	<p>Main: [1 – 9]. Additional: [10 – 14]. Methodological support: [15 – 17]</p>
	<p><i>Theme 11. Problems in the construction of linear multiple regression models.</i></p> <ol style="list-style-type: none"> <li>1. Multicollinearity and its sequences. Methods of elimination of multicollinearity.</li> <li>2. Heteroscedasticity and methods of defining heteroscedasticity. The generalized method of least squares.</li> <li>3. Autocorrelation of model residuals and methods of elimination</li> </ol>	1	<p>Main: [1 – 9]. Additional: [10 – 14]. Methodological support: [15 – 17]</p>
	<p><i>Theme 12. The generalized schemes of regression analysis.</i></p> <ol style="list-style-type: none"> <li>1. The generalized schemes of regression analysis.</li> <li>2. The generalized Aitken method of least squares. Dummy variables. Advanced methods of regression analysis</li> </ol>	1	<p>Main: [1 – 9]. Additional: [10; 12 – 14]. Methodological support: [15 – 17]</p>
	<p><i>Theme 13. The systems of econometric equations.</i></p> <ol style="list-style-type: none"> <li>1. The structure and reduced forms of a model.</li> <li>2. Estimation of parameters of a structural model.</li> <li>3. The two-stage method of least squares (2SMLS). Economic models on the basis of the system of structural equations</li> </ol>	1	<p>Main: [1 – 9]. Additional: [10; 12 – 14]. Methodological support: [15 – 17]</p>

Table 5.1 (the end)

1	2	3	4
	<i>Theme 14. Dynamic econometric models.</i> 1. Econometric models with lag variables. 2. A correlogram. The choice of a model type with separated lags. 3. Methods of partitioning of dynamic econometric models. Almon's method. Koyck's method	1	Main: [1 – 9]. Additional: [10; 12 – 14]. Methodological support: [15 – 17]
<b>Total number of hours</b>		<b>16</b>	

Conducting a practical study is based on the previously prepared material, i.e. tests designed to assess the mastery of the required theoretical statements, tasks of different complexity to be solved by students.

A practical study includes control of students' knowledge, abilities and skills, formulation of a general problem by the lecturer and discussing it with the students, solving control tasks, reviewing them, assessment.

## **5.2. Examples of typical tasks of a class written test according to the themes**

### **Thematic module 1 Optimization methods**

#### **Theme 2. Problems of linear programming and methods for solving them**

*Level 1.* The company produces articles of two kinds  $A_1$  and  $A_2$ , using raw material of three types  $S_1$ ,  $S_2$  and  $S_3$  whose respective supplies are 70, 120 and 30 per day. The expense rates of each type of raw material per one article are 7, 15, 2 for  $A_1$  and 10, 8, 5 for  $A_2$  respectively. The profit per unit output of product  $A_1$  is 4, the profit per unit output of product  $A_2$  is 2.

It is necessary to:

a) fill in the table of data:

### The table of data

Resources	Types of products		Supplies of resources
	A <sub>1</sub>	A <sub>2</sub>	
S <sub>1</sub>			
S <sub>2</sub>			
S <sub>3</sub>			
The profit per unit output of product <i>j</i>			The objective is to find a product mix maximizing the profit

- b) construct the mathematical model of this problem if the objective is to find a product mix maximizing the profit;
- c) solve it by the graphical method;
- d) draw conclusions about the outputs of product and the remains of raw materials.

*Level 2.* Using the condition of the task of the first level it is necessary to:

- a) fill in the simplex table:

### The simplex table

No.	Basis	$\overline{C}_{bas}$	$c_j$						Comments
			$\overline{A}_0$	$\overline{A}_1$	$\overline{A}_2$	$\overline{A}_3$	$\overline{A}_4$	$\overline{A}_5$	
1									
2									
3									
$z_j = \overline{C}_{bas} \cdot \overline{A}_j$									
$\Delta_j = z_j - c_j$									

- b) find an optimal solution to the initial problem with the help of the simplex table;
- c) compare the solution obtained with the help of the graphical method and the simplex method;
- d) explain the obtained results.

### Theme 3. Duality theory and analysis of linear models of economic optimization problems

Level 3. Using the condition of the task of the first level it is necessary to:

- 1) construct a dual problem for the initial primal problem;
- 2) obtain an optimal solution to the problem using duality theorems;
- 3) compare the results obtained with the help of the simplex method and duality theorems;
- 4) explain the obtained results.

## Thematic module 2 Econometric methods

### Theme 9. A pair linear model

Level 1. The results of the observations of variables  $X$  and  $Y$  are given as:

X	0.25	2.25	4.25	6.25	8.25	10.25	12.25
Y	6.0	9.5	16.0	18.0	23.5	27.0	33.0

It is necessary to:

- 1) calculate numerical characteristics (the mean, the variance, the root-mean square deviation for variables  $X$  and  $Y$ , the coefficient of covariation);
- 2) compute the empirical correlation coefficient (strength)  $r_{xy}$ , explain it;
- 3) calculate the determination coefficient  $R^2$ , explain it;
- 4) compute the elasticity coefficient:  $\bar{E}$ , explain it;
- 5) construct the theoretical regression line  $Y$  upon  $X$  as  $\hat{y}_x = b_0 + b_1x$  using the method of least squares;
- 6) explain the economic meaning of regression coefficients;
- 7) estimate the significance of the regression equation using the F-criterion (Fisher);
- 8) estimate the statistical significance of the coefficients using the t-test (Student's test);
- 9) make analysis of the constructed model.

## **Theme 10. Methods of construction of a multiple regression model**

*Level 2.* The results of 20 observations of variables  $x_1$ ,  $x_2$  and  $y$  are given as:

$$\begin{aligned} \sum x_1 &= 739; & \sum x_2 &= 180; & \sum y &= 734; & \sum x_1^2 &= 27551; & \sum x_1x_2 &= 6615; \\ \sum x_2^2 &= 1806; & \sum y^2 &= 28020; & \sum yx_1 &= 27513; & \sum yx_2 &= 6357. \end{aligned}$$

It is necessary to:

- 1) construct a linear equation of the multiple regression in natural and standardized variables;
- 2) calculate the multiple determination coefficient and the corrected coefficient, make analysis of the obtained values;
- 3) estimate the significance of the regression equation using the F-test;
- 4) estimate the significance of the regression equation using the t-test;
- 5) give the interval estimation of regression coefficients with the probability 0.95;
- 6) make analysis of the constructed model;
- 7) give prognosis intervals of  $y$  with the probability 0.95 using the expected values  $x_1 = 40$ ,  $x_2 = 5$ .

## **6. The themes of laboratory studies**

The educational plan provides conducting laboratory studies on the academic discipline "Economic and Mathematical Methods" in the first term.

**A laboratory study** is a form of study when a student under the direction of a lecturer fulfills a practical task with the help of PC-programming (software MatLab). The plan of laboratory studies, their content and bibliography for each theme are given in Table 6.1.

Conducting a laboratory study on the defined theme is preceded by analysis of the basic theoretical fundamentals forming practical skills. A laboratory study is fulfilled in the computer room with the use of MS Excel. It favours the following: firstly, a student extends the knowledge of the basic formulas and relations fulfilling calculations by direct writing the corresponding formulas and, secondly, acquires skills in the use of built-in functions of MS Excel. This kind of approach gives a possibility to pay more attention to economic explanation of mathematical transformations.



Table 6.1

### The plan of the themes of laboratory studies

The theme name	The syllabus questions	Hours	Recommended reading
1	2	3	4
<b>Thematic module 1 Optimization methods</b>			
<i>Theme 1.</i> Built-in functions of MS Excel. Elements of linear algebra in MS Excel	Learning the programming software MS Excel. Basic built-in functions of MS Excel which are used in economic and mathematical methods	2	Main: [1 – 9]. Additional: [10; 13]. Methodological support: [15 – 17]
<i>Theme 2.</i> Simplex method of solving problems of linear optimization	Solving linear programming problems with the help of the simplex method based on the example of product mix problems using built-in functions of MS Excel. Solving problems using the built-in function "Solver Add-in"	2	Main: [1 – 9]. Additional: [10; 13]. Methodological support: [15 – 17]
<i>Theme 3.</i> The transportation problem	Solving transportation problems using the built-in function "Solver Add-in" of MS Excel. Investigation of stability of an optimal solution relative to supplies and demands, costs and transportation of a unit of goods	2	Main: [1 – 9]. Additional: [10; 13]. Methodological support: [15 – 17]
<i>Theme 4.</i> Forming an investment portfolio as a quadratic programming problem	Investigation of a conditional extremum of problems with the help of Lagrange's function. Solving a problem of an investment portfolio as a quadratic programming problem. Construction of a mathematical model of a pair game with a zero sum as a linear programming problem. Using built-in functions of MS Excel for solving a two-person game	2	Main: [1 – 9]. Additional: [10; 13]. Methodological support: [15 – 17]

Table 6.1 (continuation)

1	2	3	4
<b>Thematic module 2</b> <b>Econometric methods</b>			
<i>Theme 5.</i> A linear pair regression model. Checking the significance of parameters of a pair regression model	Using the observations $(X, Y)$ it is necessary to: 1) estimate the parameters of the linear model $\hat{y}_x = b_0 + b_1x$ ; 2) construct a theoretical regression line and its 95 % confidence interval; 3) explain the results	2	Main: [1 – 9]. Additional: [10; 12 – 14]. Methodological support: [15 – 17]
<i>Theme 6.</i> A multifactor linear model	Using the observations $(X_1, X_2, X_3, Y)$ it is necessary to: 1) calculate the parameters of a linear model in a matrix form; 2) calculate the parameters using built-in functions of MS Excel; 3) fill in the table of variance analysis for $m \leq 3$	2	Main: [1 – 9]. Additional: [10 – 14]. Methodological support: [15 – 17]
<i>Theme 7.</i> Investigation of problems of a linear multifactor model	Using the observations $(X_1, X_2, X_3, X_4, Y)$ it is necessary to: 1) define the estimators of the parameters of a multifactor model; 2) compute the calculated value $Y_{calc}$ changing the explained variable $X$ with fixed values of author factors; 3) plot graphs of values according to each argument; 4) check the stability of the calculated estimators; 5) define the fact of multicollinearity; 6) write down a regression equation and explain each parameters; 7) calculate the value of the determination coefficient and explain it; 8) define the significance of the model using the Fisher test;	2	Main: [1 – 9]. Additional: [10 – 14]. Methodological support: [15 – 17]

Table 6.1 (the end)

1	2	3	4
	9) estimate the significance of each parameter of the model using Student's test; 10) construct the confidence intervals of the regression coefficients		
<i>Theme 8. Confidence limits for a regression line. Confidence limits in multiple regression</i>	Using the observations $(X_1, X_2, X_3, Y)$ it is necessary to: 1) plot graphs of component effects with 95 % confidence intervals of the calculated values $Y_{calc}$ and expected data scattering around the regression line; 2) calculate the value of the determination coefficient and explain it; 3) define the significance of the model using the Fisher test 4) construct the confidence intervals of the regression coefficients	2	Main: [1 – 9; 14]. Additional: [10 – 13]. Methodological support: [15 – 17]
<b>Total number of hours</b>		<b>16</b>	

## 7. Independent work

### 7.1. Forms of independent work

Independent work is a scheduled educational and scientific work which is carried out on a lecture task under the methodical and scientific guidance of a lecturer. It is a specific form of the educational activity; its main objective is to form independence of a person.

Independent work is:

1) different forms of individual and group cognitive activity of students, which is fulfilled by them during practical studies and in the extracurricular time;

2) different types of educational tasks which are fulfilled under the guidance of a lecturer;

3) a system of work organization when management of the educational work of students is fulfilled in the absence of a lecturer and without his direct assistance;

4) work of students which is carried out according to a specific individual educational plan designed on the basis of taking into account individual characteristics and cognitive possibilities of students.

The types of independent work and forms of control are given in Table 7.1.

The educational time, which is intended for students' independent work of the day-time form of study, is defined according to the educational plan and makes 60 % (90 hours) out of the total educational time for learning the discipline. For students of the distant form of study this time equals 82 % (148 hours) out of the total educational time for learning the discipline.

During independent work students become active participants in the educational process, learn to master consciously the theoretical and practical material, orientate easily in the information space, take responsibility for the quality of their professional training.

Table 7.1

### Tasks for students' independent work and forms of control

The name of the theme	The content of students' independent work	The number of hours	Forms of control of IWS	Recommended reading
1	2	3	4	5
<b>Thematic module 1</b>				
<b>Optimization methods</b>				
<b>Theme 1. The general theoretical foundations of optimization methods and models in economics</b>	Learning the lecture material. Preparation for a practical study and laboratory work. Preparation for the defence of the laboratory work. Independent learning of the questions: a) fulfillment of the statement and formalization of practical tasks according to a general technology of modelling in economics; b) classification of models of mathematical programming problems and choosing the mathematical methods for solving them. Carrying out homework and independent work	5	Homework	Main: [1 – 4; 6; 9]. Additional: [10; 13]. Methodological support: [15 – 17]

Table 7.1 (continuation)

1	2	3	4	5
<p><b>Theme 2.</b> <b>Problems of linear programming and methods for solving them</b></p>	<p>Learning the lecture material. Preparation for a practical study and laboratory work. Preparation for the defence of the laboratory work. Independent learning of the questions: a) solving economic problems with the help of the methods of linear programming. Carrying out homework and independent work. Preparation for an independent test</p>	5	Homework	<p>Main: [1 – 4; 6; 9]. Additional: [10; 13]. Methodological support: [15 – 17]</p>
<p><b>Theme 3.</b> <b>Duality theory and analysis of linear models of economic optimization problems</b></p>	<p>Learning the lecture material. Preparation for a practical study and laboratory work. Preparation for the defence of the laboratory work. Independent learning of the questions: a) construction of a model of a dual problem; b) defining a solution to the initial problem according to the solution to a dual problem and giving an economic explanation of dual estimations. Carrying out homework and independent work. Preparation for an independent test</p>	4	Homework	<p>Main: [1 – 4; 6; 9]. Additional: [10; 13]. Methodological support: [15 – 17]</p>
<p><b>Theme 4. The transportation problem</b></p>	<p>Learning the lecture material. Preparation for a practical study and laboratory work. Preparation for the defence of the laboratory work. Independent learning of the questions: a) obtaining the initial transportation (solution) using different methods;</p>	5	Homework	<p>Main: [1 – 4; 6; 9]. Additional: [10; 13]. Methodological support: [15 – 17]</p>

Table 7.1 (continuation)

1	2	3	4	5
	<p>b) solving transportation problems with the help of the method of potentials.</p> <p>Carrying out homework and independent work.</p> <p>Preparation for an independent test, a competence-oriented task.</p> <p>Preparation for the presentation of an independent creative task</p>			
<p><b>Theme 5.</b> <b>Integer programming</b></p>	<p>Learning the lecture material.</p> <p>Preparation for a practical study and laboratory work.</p> <p>Preparation for the defence of the laboratory work.</p> <p>Independent learning of the questions:</p> <p>a) cutting methods;</p> <p>b) combinatorial methods;</p> <p>c) methods of approximate computations;</p> <p>d) the branch and bound method;</p> <p>e) Gomory method (the cutting method).</p> <p>Carrying out homework and independent work.</p> <p>Preparation for the presentation of an independent creative task.</p> <p>Preparation for the colloquim, a written test and a competence oriented task</p>	4	Homework. An independent test	<p>Main: [1 – 4; 6; 9].</p> <p>Additional: [10; 13].</p> <p>Methodological support: [15 – 17]</p>
<p><b>Theme 6.</b> <b>Nonlinear optimization models of economic systems</b></p>	<p>Learning the lecture material.</p> <p>Preparation for a practical study and laboratory work.</p> <p>Preparation for the defence of the laboratory work.</p> <p>Independent learning of the questions:</p> <p>a) solving separate problems of nonlinear programming using the graphical method and the method of Lagrange's multipliers</p>	5	Homework. A competence-oriented task	<p>Main: [1 – 4; 6; 9].</p> <p>Additional: [10; 13].</p> <p>Methodological support: [15 – 17]</p>

Table 7.1 (continuation)

1	2	3	4	5
	Carrying out homework and independent work. Preparation for an independent test and a written test Preparation for the presentation of an independent creative task and the colloquium			
<b>Theme 7. Game theory. Analysis and risk management in economics on the basis of the concept of game theory</b>	Learning the lecture material. Preparation for a practical study and laboratory work. Preparation for the defence of the laboratory work. Independent learning of the questions: a) using the elements of game theory for solving pair matrix games with a zero sum. Carrying out homework and independent work. Preparation for an independent test. Preparation for the presentation of an independent creative task	6	Homework. A written test on the themes. A colloquium on themes 1 – 7	Main: [1 – 4; 6; 9]. Additional: [10; 13]. Methodological support: [15 – 17]
<b>Total for thematic module 1</b>		<b>34</b>		
<b>Thematic module 2 Econometric methods</b>				
<b>Theme 8. Particular properties of construction of econometric models and ways of construction</b>	Learning the lecture material. Preparation for a practical study and laboratory work. Preparation for the defence of the laboratory work. Independent learning of the questions: a) particular properties and ways of construction of econometric models. Carrying out homework and independent work. Preparation for an independent test. Preparation for the presentation of an independent creative task	4	Homework	Main: [1 – 9]. Additional: [10 – 14]. Methodological support: [15 – 17]

Table 7.1 (continuation)

1	2	3	4	5
<p><b>Theme 9.</b> <b>A pair linear model</b></p>	<p>Learning the lecture material. Preparation for a practical study and laboratory work. Preparation for the defence of the laboratory work. Independent learning of the questions: a) calculation of numerical characteristics of dependent and independent factors; b) construction of a pair linear regression equation. Carrying out homework and independent work. Preparation for an independent test. Preparation for the presentation of an independent creative task</p>	6	Homework	<p>Main: [1 – 9]. Additional: [10 – 14]. Methodological support: [15 – 17]</p>
<p><b>Theme 10. Methods of construction of a multiple regression model</b></p>	<p>Learning the lecture material. Preparation for a practical study and laboratory work. Preparation for the defence of the laboratory work. Independent learning of the questions: a) the least squares method (MLS); b) statistical properties of MLS estimators; c) variance and standard errors of parameters of an equation. Carrying out homework and independent work. Preparation for an independent test. Preparation for the presentation of an independent creative task</p>	7	Homework	<p>Main: [1 – 9]. Additional: [10 – 14]. Methodological support: [15 – 17]</p>



Table 7.1 (continuation)

1	2	3	4	5
<p><b>Theme 11.</b> <b>Problems in the construction of linear multiple regression models</b></p>	<p>Learning the lecture material. Preparation for a practical study and laboratory work. Preparation for the defence of the laboratory work. Independent learning of the questions: a) multicollinearity and its sequences; b) heteroscedasticity and methods of defining of heteroscedasticity; c) autocorrelation of model residuals and methods of elimination; d) estimation of parameters of a model with autoregression. Carrying out homework and independent work. Preparation for an independent test and the colloquium. Preparation for the presentation of an independent creative task</p>	7	Homework	<p>Main: [1 – 9]. Additional: [10 – 14]. Methodological support: [15 – 17]</p>
<p><b>Theme 12.</b> <b>The generalized schemes of regression analysis</b></p>	<p>Learning the lecture material. Preparation for a practical study and laboratory work. Preparation for the defence of the laboratory work. Independent learning of the questions: a) generalized schemes of regression analysis; b) dummy variables. Carrying out homework and independent work. Preparation for an independent test, the colloquium, a written test and a competence-oriented task. Preparation for the presentation of an independent creative task</p>	7	Homework	<p>Main: [1 – 9]. Additional: [10 – 14]. Methodological support: [15 – 17]</p>

Table 7.1 (continuation)

1	2	3	4	5
<p><b>Theme 13.</b> <b>The systems of econometric equations</b></p>	<p>Learning the lecture material. Preparation for a practical study and laboratory work. Preparation for the defence of the laboratory work. Independent learning of the questions: a) systems of econometric equations; b) the two-stage method of least squares of solving systems of econometric equations. Carrying out homework and independent work. Preparation for an independent test, preparation for the colloquim, a written test and a competence oriented task. Preparation for the presentation of an independent creative task</p>	6	Homework. An independent test	<p>Main: [1 – 9]. Additional: [10 – 14]. Methodological support: [15 – 17]</p>
<p><b>Theme 14.</b> <b>Dynamic econometric models</b></p>	<p>Learning the lecture material. Preparation for a practical study and laboratory work. Preparation for the defence of the laboratory work. Independent learning of the questions: a) the concepts of lag and lag variables; b) the general characteristics of models with separated lags; c) the types of lag models; d) the meaning of parameters of models with a separated lag; e) defining the lag structure; f) lags of independent variables; g) a mutual correlation function; h) a correlogram</p>	5	<p>A competence-oriented task. A written test. A colloquim on themes 8 – 14. An independent creative task</p>	<p>Main: [1 – 9]. Additional: [10 – 14]. Methodological support: [15 – 17]</p>

Table 7.1 (the end)

1	2	3	4	5
	i) Almon's method; j) Koyck's method. Carrying out homework and an independent test			
<b>Total for thematic module 2</b>		<b>42</b>		
<i>Preparation for the exam</i>		<b>10</b>	Exam	Main: [1 – 9]. Additional: [10 – 14]. Methodological support: [15 – 17]
<i>Preexam consultations</i>		<b>2</b>		
<i>Exam</i>		<b>2</b>		
<b>Total for the academic discipline</b>		<b>90</b>	<b>–</b>	<b>–</b>

The necessary element of successful mastering of the academic discipline material is the students' independent work (SIW) on specific mathematical and economic literature.

SIW includes: processing the lecture material (a lecture as a form of education provides theoretical knowledge besides being used for carrying out practical calculations); processing and learning the recommended literature, basic terms and concepts on the themes of the academic discipline; preparation for practical and laboratory studies; preparation for the defence of the laboratory work; advanced study of particular themes or questions of lectures; carrying out practical homework, solving computational competence-oriented tasks on the given theme; choosing and consideration of literature sources on the given problem of the academic discipline; analytic consideration of scientific publications; self-assessment of students' knowledge based on the questions for self-assessment; carrying out independent work; carrying out independent creative work; preparation for tests and other forms of current control; preparation for module control (a colloquium); systematization of the studied material for the purpose of preparation for term exams on each module of the academic discipline.

## 7.2. Examples of practical homework for independent work

### Thematic module 1 Optimization methods

*Theme 1. The general theoretical foundations of optimization methods and models in economics.*

**Task 1.1.** The company produces articles of two kinds  $A_1$  and  $A_2$ , using raw material of three types  $S_1$ ,  $S_2$  and  $S_3$  whose respective supplies are 10, 16 and 12 per day. The expense rates of each type of raw material per one article are 3, 2, 4 for  $A_1$  and 2, 8, 2 for  $A_2$  respectively. The profit per unit output of product  $A_1$  is 12, the profit per unit output of product  $A_2$  is 9. Construct a mathematical model of this problem if the objective is to find a product mix maximizing the profit.

**Task 1.2.** The animal diet includes feeds of two kinds  $A_1$  and  $A_2$ . Animals must receive nutrients of three types daily ( $S_1$ ,  $S_2$  and  $S_3$ ). The amount of nutrient  $j$  must be at least  $b_j$ . Let  $a_{ij}$  be the content of nutrient  $i$  per unit of feed  $j$ , and let  $c_j$  be the cost per unit of feed  $j$ . Construct a mathematical model of this problem if the objective is to find a diet minimizing the daily cost.

1)  $b_1 = 12$ ,  $b_2 = 10$ ,  $b_3 = 24$ ,  $a_{11} = 2$ ,  $a_{21} = 1$ ,  $a_{31} = 2$ ,  $a_{12} = 1$ ,  $a_{22} = 1$ ,  $a_{32} = 3$ ,  $c_1 = 60$ ,  $c_2 = 60$ ;

2)  $b_1 = 15$ ,  $b_2 = 12$ ,  $b_3 = 7$ ,  $a_{11} = 5$ ,  $a_{21} = 2$ ,  $a_{31} = 1$ ,  $a_{12} = 1$ ,  $a_{22} = 1$ ,  $a_{32} = 1$ ,  $c_1 = 40$ ,  $c_2 = 30$ .

**Task 1.3.** The company produces articles of two kinds  $A_1$  and  $A_2$ , using raw material of three types  $S_1$ ,  $S_2$  and  $S_3$  whose respective supplies are 100, 180 and 100 per day. The expense rates of each type of raw material per one article are 0.2, 0.2, 0.1 for  $A_1$  and 0.1, 0.5, 0.2 for  $A_2$  respectively. The profit per unit output of product  $A_1$  is 10, the profit per unit output of product  $A_2$  is 16. Construct a mathematical model of this problem if the objective is to find a product mix maximizing the profit on the condition: the company needs to produce no more than 300 articles  $A_1$  and no more than 200 articles  $A_2$ .

**Task 1.4.** Find extremums of the functions:

1)  $y = \frac{1}{x_1} + \frac{1}{x_2}$  under the condition  $x_1 + x_2 = 2$ ;

2)  $y = x_1 + x_2$  under the condition  $\frac{1}{x_1^2} + \frac{1}{x_2^2} = \frac{1}{2}$ ;

3)  $y = x_1 x_2$  under the condition  $x_1^2 + x_2^2 = 2$ .

**Task 1.5.** Using Lagrange method, define stationary points in the problems of investigation of the conditional extremum:

1)  $z = \sqrt{x_1^2 + x_2^2}$  if  $3x_1 + 4x_2 \leq 24$ ,  $x_1 \geq 0$ ,  $x_2 \geq 0$ ;

2)  $z = (x_1 - 2)^2 + (x_2 - 3)^2$  if  $x_1 + 2x_2 \leq 12$ ,  $x_1 + x_2 \leq 9$ ,  $x_1 \geq 0$ ,  $x_2 \geq 0$ ;

3)  $z = x_1^2 - x_2^2$  if  $3x_1 + 4x_2 = 12$ ;

4)  $z = x_1^2 + 2x_1 x_2 + 2x_2^2$  if  $4x_1^2 + x_2^2 = 25$ ;

5)  $z = 2x_1 + 3x_2 - 2x_1^2$  if  $x_1 + 2x_2 \leq 4$ ,  $x_1 + x_2 \leq 2$ ,  $x_1 \geq 0$ ,  $x_2 \geq 0$ ;

6)  $z = x_1 - 2x_2 + 2x_3$  if  $x_1^2 + x_2^2 + x_3^2 = 1$ .

**Task 1.6.** For the given problems find a solution with the help of the graphical method, write down Lagrange's function and find its saddle point:

1)  $g = 2(x_1 - 5)^2 + (x_2 - 7)^2 \rightarrow \min$ ,    2)  $g = (x_1 - 4)^2 + (x_2 - 6)^2 \rightarrow \max$ ,

$$\begin{cases} x_1 + x_2 \leq 9, \\ x_1 + 2x_2 \leq 19, \end{cases}$$

$$x_1 \geq 0, x_2 \geq 0;$$

$$\begin{cases} x_1 + x_2 \geq 1, \\ 2x_1 + 3x_2 \leq 12, \end{cases}$$

$$x_1 \geq 0, x_2 \geq 0;$$

3)  $g = 2(x_1 - 7)^2 + 4(x_2 - 3)^2 \rightarrow \max$ ,    4)  $g = -x_1 - 2x_2 + x_2^2 \rightarrow \min$ ,

$$\begin{cases} x_1 + 2x_2 \geq 2, \\ x_1 + x_2 \leq 6, \\ 2x_1 + x_2 \leq 10, \end{cases}$$

$$x_1 \geq 0, x_2 \geq 0;$$

$$\begin{cases} 3x_1 + 2x_2 \leq 6, \\ x_1 + 2x_2 \leq 4, \end{cases}$$

$$x_1 \geq 0, x_2 \geq 0.$$

*Theme 2. Problems of linear programming and methods for solving them.*

**Task 2.1.** The company produces articles of two kinds  $A_1$  and  $A_2$ , using raw material of three types  $S_1$ ,  $S_2$  and  $S_3$  whose respective supplies are 20, 12 and 30 per day. The expense rates of each type of raw material per one article are 2, 1, 1 for  $A_1$  and 1, 1, 3 for  $A_2$  respectively. The profit per unit output of product  $A_1$  is 4, the profit per unit output of product  $A_2$  is 5.

Do the following:

- construct a mathematical model of this problem if the objective is to find a product mix maximizing the profit;
- solve it by the graphical method;
- solve it by the simplex method.

**Task 2.2.** The animal diet includes feeds of two kinds  $A_1$  and  $A_2$ . Animals must receive nutrients of three types daily ( $S_1$ ,  $S_2$  and  $S_3$ ). The amount of nutrient  $j$  must be at least  $b_j$ . Let  $a_{ij}$  be the content of nutrient  $i$  per unit of feed  $j$ , and let  $c_j$  be the cost per unit of feed  $j$ .

$$1) b_1 = 5, b_2 = 500, b_3 = 20, a_{11} = 1, a_{21} = 80, a_{31} = 1, a_{12} = 0,5, a_{22} = 200, a_{32} = 8, c_1 = 3, c_2 = 5;$$

$$2) b_1 = 1, b_2 = 5, b_3 = 400, a_{11} = 0.1, a_{21} = 1, a_{31} = 110, a_{12} = 0.25, a_{22} = 0.25, a_{32} = 120, c_1 = 3.8, c_2 = 4.2.$$

It is necessary to:

- construct a mathematical model of this problem if the objective is to find a diet minimizing the daily cost;
- solve it by the graphical method;
- solve it by the simplex method.

**Task 2.3.** Solve the problems by the graphical method:

$$1) \quad z = 2x_1 - 3x_2 \rightarrow \min$$

$$\begin{cases} x_1 + x_2 \geq 4, \\ 2x_1 - x_2 \geq 1, \\ x_1 - 2x_2 \leq 1, \\ x_1 \geq 0, x_2 \geq 0. \end{cases}$$

$$2) \quad z = 2x_1 + 5x_2 \rightarrow \max;$$

$$\begin{cases} 4x_1 + 5x_2 \leq 40, \\ 2x_1 + 3x_2 \geq 6, \\ -x_1 + x_2 - 5 \leq 0, \\ x_1 \geq 0, x_2 \geq 0. \end{cases}$$

3)

$$z = x_1 + 3x_2 \quad (\text{max}),$$

$$\begin{cases} x_1 - x_2 \leq 1, \\ 2x_1 + x_2 \leq 2, \\ x_1 - x_2 \geq 0, \end{cases}$$

$$x_1 \geq 0; \quad x_2 \geq 0.$$

4)

$$z = 2x_1 - x_2 \quad (\text{min}),$$

$$\begin{cases} x_1 + x_2 \geq 4, \\ -x_1 + 2x_2 \leq 2, \\ x_1 + 2x_2 \leq 10, \end{cases}$$

$$x_1 \geq 0, x_2 \geq 0.$$

5)

$$z = x_1 - 2x_2 \quad (\text{max}),$$

$$\begin{cases} x_1 - x_2 \leq 1, \\ x_1 + x_2 \geq 2, \\ x_1 - 2x_2 \leq 0, \end{cases}$$

$$x_1 \geq 0, \quad x_2 \geq 0.$$

6)

$$z = x_1 - 2x_2 \quad (\text{min})$$

$$\begin{cases} x_1 + x_2 - 1 \geq 0, \\ x_1 - x_2 + 2 \geq 0, \\ -x_1 + 2x_2 + 2 \geq 0, \\ -x_1 + 3 \geq 0, \\ x_1 \geq 0, \quad x_2 \geq 0. \end{cases}$$

7)

$$z = 2x_1 + 2x_2 \rightarrow \max$$

$$\begin{cases} 3x_1 - 2x_2 \geq -6, \\ x_1 + x_2 \geq 3, \\ 0 \leq x_1 \leq 3, \\ 0 \leq x_2 \leq 5. \end{cases}$$

8)

$$z = -3x_1 + x_2 \rightarrow \min$$

$$\begin{cases} x_1 + 2x_2 \geq 10, \\ 3x_1 + x_2 \geq 15, \\ x_1 \leq 8, \\ x_1 \geq 0, x_2 \geq 0. \end{cases}$$

**Task 2.4.** Reduce the LP problems to the standard form:

1)

$$z = x_1 - 3x_2 + 2x_3 \rightarrow \min$$

subject to

$$\begin{cases} 2x_1 - x_3 \leq 2 \\ x_1 + 2x_2 + x_3 \geq 5 \\ x_j \geq 0 \quad j = 1, \dots, 3 \end{cases}$$

2)

$$z = 2x_1 + x_2 \rightarrow \max$$

subject to

$$\begin{cases} x_1 + x_2 \geq 1 \\ -2x_1 + 3x_2 \leq 16 \\ 4x_1 + x_2 \leq 12 \\ x_j \geq 0 \quad j = 1, 2 \end{cases}$$

**Task 2.5.** Construct a mathematical model and solve this problem by the simplex method.

Resources	Types of product		Supplies of resources
	$A_1$	$A_2$	
$S_1$	1	0	10
$S_2$	0	2	30
$S_3$	1	2	47
The profit per unit output of product $j$	40	70	The objective is to find a product mix maximizing the profit

**Task 2.6.** Construct a mathematical model and solve this problem by the simplex method.

Nutrients	Types of feeds		Amount of nutrient
	$A_1$	$A_2$	
$S_1$	3	1	9
$S_2$	1	2	8
$S_3$	1	6	12
The cost per unit of feed $j$	4	6	The objective is to find a diet minimizing the total cost

**Task 2.7.** Construct a mathematical model and solve this problem by the simplex method.

Resources	Types of product			Supplies of resources
	$A_1$	$A_2$	$A_3$	
$S_1$	1	6	3	84
$S_2$	3	1	3	42
$S_3$	1	3	2	21
$S_4$	2	3	4	42
The profit per unit output of product $j$	3	6	4	The objective is to find a product mix maximizing the profit



**Task 2.8.** Construct a mathematical model and solve this problem by the simplex method.

Nutrients	Feeds			Amount of nutrient
	$A_1$	$A_2$	$A_3$	
$S_1$	50	20	180	2000
$S_2$	6	4	3	120
$S_3$	2	1	1	40
The cost per unit of feed $j$	30	20	50	The objective is to find a diet minimizing the total cost

*Theme 3. Duality theory and analysis of linear models of economic optimization problems.*

**Task 3.1.** Construct a dual problem and solve it using the first and the second duality theorems if the primal problem has the form:

$$z = -2x_1 + 4x_2 + 14x_3 + 2x_4 \rightarrow \min$$

$$\begin{cases} -2x_1 - x_2 + x_3 + 2x_4 \geq 6, \\ x_1 - 2x_2 - 4x_3 + 5x_4 \leq -30, \\ x_j \geq 0 \quad j = 1, \dots, 4. \end{cases}$$

and its optimal solution is  $X_{opt} = (0, 1, 7, 0)$  with  $z_{\min} = 102$ .

**Task 3.2.** Construct a dual problem and solve it using the first and the second duality theorems if the primal problem has the form:

$$z = 2x_1 + 3x_2 + 2x_3 + x_4 \rightarrow \max$$

$$\begin{cases} 2x_1 + 2x_2 - 3x_3 + x_4 \leq 6, \\ x_2 - x_3 + x_4 \leq 2, \\ x_1 - x_2 + 2x_3 \leq 5, \\ x_j \geq 0 \quad j = 1, \dots, 4. \end{cases}$$

and its optimal solution is  $X_{opt} = (0, 9, 7, 0)$  with  $z_{\max} = 41$ .

**Task 3.3.** Construct a dual problem and solve it using the first and second duality theorems if the primal problem has the form:

$$z = 4x_1 + 6x_2 \rightarrow \min$$

$$\begin{cases} 3x_1 + x_2 \geq 9, \\ x_1 + 2x_2 \geq 8, \\ x_1 + 6x_2 \geq 12, \\ x_1 \geq 0, x_2 \geq 0. \end{cases}$$

and its optimal solution is  $X_{opt} = (2; 3)$  with  $z_{\min} = 26$ .

*Theme 4. The transportation problem.*

**Task 4.1.** Find the initial solution and  $Z(X)$  of the transportation problem using supply source  $a_i$  and consumer destination  $b_j$  with the matrix  $C$  of the transportation cost:

- with the help of the minimal cost method;
- with the help of the north-west corner method.

$$1) \begin{matrix} a_1 = 80, & b_1 = 45, \\ a_2 = 50, & b_2 = 35, \\ a_3 = 70, & b_3 = 55, \\ & b_4 = 65, \end{matrix} \quad C = \begin{pmatrix} 4 & 1 & 2 & 3 \\ 3 & 2 & 3 & 7 \\ 4 & 4 & 5 & 2 \end{pmatrix};$$

$$2) \begin{matrix} a_1 = 180, & b_1 = 120, \\ a_2 = 40, & b_2 = 40, \\ a_3 = 80, & b_3 = 60, \\ & b_4 = 80, \end{matrix} \quad C = \begin{pmatrix} 2 & 3 & 4 & 3 \\ 5 & 3 & 1 & 2 \\ 2 & 1 & 4 & 2 \end{pmatrix}.$$

3)

$A_i \backslash B_j$	$B_1$	$B_2$	$B_3$	$B_4$	$a_i$
$A_1$	2	5	8	1	9
$A_2$	8	3	9	2	16
$A_3$	7	4	6	3	5
$b_j$	11	7	8	4	

4)

$A_i \backslash B_j$	$B_1$	$B_2$	$B_3$	$B_4$	$B_5$	$a_i$
$A_1$	4	6	8	3	2	7
$A_2$	5	3	4	6	4	13
$A_3$	3	2	5	7	5	20
$b_j$	10	10	5	8	7	

## Thematic module 2 Econometric methods

*Theme 9. A pair linear model.*

**Task 9.1.** The results of observations of variables  $X$  and  $Y$  are given as:

a) 

$X$	14	16	18	20	22
$Y$	15	18	17	19	24

b) 

$X$	1.5	3.0	4.5	6.0	7.5	9.0	10.5
$Y$	14.0	24.0	33.0	37.0	46.0	51.0	63.0

c) 

$X$	0.25	2.25	4.25	6.25	8.25	10.25	12.25
$Y$	6.0	9.5	16.0	18.0	23.5	27.0	33.0

d) 

$X$	2.0	3.5	4.0	5.5	7.0	8.5	9.0
$Y$	10	21	25	33	41	51	63

e) 

$X$	0.5	3.5	6.5	9.5	12.5	15.5	18.5
$Y$	52.3	48.2	43.5	40.2	38.3	35.2	30.1

f) 

$X$	3.25	4.25	9.25	12.25	13.25	18.25	21.25
$Y$	22.0	20.0	17.0	15.5	12.5	10.2	8.8

Do the following:

- 1) calculate the coefficients of the pair linear regression;
- 2) construct a theoretical regression line  $Y$  upon  $X$  ;
- 3) calculate the correlation coefficient;
- 4) calculate the determination coefficient;
- 5) calculate the standard error of the regression;
- 6) calculate the standard errors of the coefficients of the regression;
- 7) estimate the statistical significance of the coefficients;
- 8) estimate the quality of the constructed regression line;
- 9) make analysis of the constructed models.

**Task 9.2.** The results of observations of variables  $X$  and  $Y$  are given as:

The number of employees	Average productivity, $X$ , \$	Average pay, $Y$ , \$	Standard deviation of pay, \$
4	9 320	3 320	740
9	8 630	3 640	850
18	8 050	3 900	730
48	9 320	4 120	820
89	8 600	4 090	950
159	9 120	4 200	1 100
319	9 540	4 380	1 250
899	9 730	4 500	1 290
1 569	10 120	4 610	1 350
3 559	10 740	4 800	1 100
6 000	11 200	5 000	1 520

Construct a regression equation  $Y = b_0 + b_1X + e$  using the method of least squares.

*Theme 10. Methods of construction of a multiple regression model.*

**Task 10.1.** The data are given as: 1)  $n = 20$ ,  $\sum x_{1i} = 4.88$ ,  $\sum x_{2i} = 26.7$ ,  $\sum x_{1i}^2 = 2.518$ ,  $\sum x_{2i}^2 = 75.15$ ,  $\sum y_i = 44.7$ ,  $\sum x_{1i}x_{2i} = 13.75$ ,  $\sum x_{1i}y_i = 22.1$ ,  $\sum x_{2i}y_i = 125.75$ ,  $\sum y_i^2 = 210.4$ ,  $\sum (y_i - \tilde{y}_i)^2 = 0.015$ ;

2)  $n = 20$ ,  $\sum x_{1i} = 739$ ,  $\sum x_{2i} = 180$ ,  $\sum x_{1i}^2 = 27551$ ,  $\sum x_{2i}^2 = 1806$ ,  $\sum y_i = 734$ ,  $\sum x_{1i}x_{2i} = 6615$ ,  $\sum x_{1i}y_i = 27513$ ,  $\sum x_{2i}y_i = 6357$ ,  $\sum y_i^2 = 28020$ .

Do the following:

- calculate basic numerical characteristics (the mean, the variance, the root mean square deviation);
- construct an econometric model in standardized coefficients. Draw a conclusion about the influence of  $x_1$ ,  $x_2$  on  $y$ ;
- construct an econometric model in natural coefficients;
- estimate the significance of the regression equation with the help of the Fisher test.

**Task 10.2.** The data are given in the table ( $n = 30$ ).

1)

	Mean	Root-mean square deviation	Pair correlation coefficient
$y$	100	25	$r_{yx_1} = 0.67$
$x_1$	300	45	$r_{yx_2} = 0.54$
$x_2$	36.7	20	$r_{x_1x_2} = 0.23$

2)

	Mean	Root-mean square deviation	Pair correlation coefficient
$y$	250	38	$r_{yx_1} = 0.68$
$x_1$	47	12	$r_{yx_2} = 0.63$
$x_2$	112	21	$r_{x_1x_2} = 0.42$

Do the following:

a) construct an econometric model in standardized coefficients and an econometric model in natural coefficients;

b) estimate the significance of the regression equation with the help of the Fisher test.

**Task 10.3.** The matrix of pair correlation coefficients is given in the table ( $n = 25, n = 20$ ).

1)

	$y$	$x_1$	$x_2$
$y$	1	0.6	0.5
$x_1$	0.6	1	0.9
$x_2$	0.5	0.9	1

2)

	$y$	$x_1$	$x_2$
$y$	1	0.3	0.6
$x_1$	0.3	1	0.8
$x_2$	0.6	0.8	1

Do the following:

a) construct an econometric model in standardized coefficients. Draw a conclusion about the influence of  $x_1, x_2$  on  $y$ ;

b) calculate the partial correlation coefficients with each of the factors;

c) compare them with the pair correlation coefficients, draw a conclusion;

d) estimate the significance of the regression equation with the help of the Fisher test and Student's t-test.

**Task 10.4.** Construct an equation of multiple regression in natural variables if

$$n = 20, \sum x_{1i} = 62.8, \sum x_{2i} = 54.4, \sum y_i = 383.2, \beta_1 = 0.838, \\ \beta_2 = 0.557, \sigma_{x_1}^2 = 0.98, \sigma_{x_2}^2 = 4.02, \sigma_y^2 = 10.8.$$

Estimate the significance of the regression equation with the help of the Fisher test and Student's t-test.

**Task 10.5.** The data are given in the table.

$y$	14	16	18	20	23	23.5	25	26.5	28.5	30.5
$x_1$	1.65	1.8	2.0	2.1	2.2	2.4	2.65	2.85	3.2	3.55
$x_2$	8	9,5	11	12	13	14	15	16,5	17	18

Do the following:

- calculate basic numerical characteristics (the mean, the variance, the root mean square deviation);
- construct an econometric model in standardized coefficients and an econometric model in natural coefficients;
- calculate the theoretical value  $\tilde{y}(3.8; 22)$ ;
- estimate the significance of the regression equation with the help of the Fisher test and Student's t-test.

### 7.3. Questions for self-assessment

#### Thematic module 1 Optimization methods

##### Theme 1. The general theoretical foundations of optimization methods and models in economics

- An economic mathematical model.
- Construction of economic mathematical models.
- The place of modelling among the methods of learning social economic systems.
- The objective of modelling.
- Defining models, properties of models.

6. The choice of the method for solving economic mathematical problems and an explanation of this choice.

7. Possibilities of realization of the choice of a solution to optimization problems with the help of the package of applied programs for personal computer.

8. The foundations of the classical optimization theory.

9. General remarks.

10. Classification of problems.

11. Information support for economic and mathematical optimization models.

12. The statement of an optimization problem.

13. The conditional extremum.

14. Lagrange's method of multipliers.

15. The economic meaning of Lagrange's method of multipliers.

16. The iterative method of solving problems of mathematical programming.

## **Theme 2. Problems of linear programming and methods for solving them**

1. A linear programming problem (LPP).

2. Economic statements of LPP.

3. The mathematical statement of LPP.

4. The system of hypothesis.

5. A standard form of a linear optimization model.

6. A set of feasible solutions and an optimal solution to LPP.

7. The graphical method of solving LPP.

8. Possibilities of the graphical method of solving LPP.

9. The geometrical meaning of LPP.

10. Examples of problems which can be solved by the graphical method.

11. The simplex method of solving LPP.

12. The canonical (basic) form of LPP.

13. Construction (plotting) of support solutions.

14. The optimization criterion.

15. Searching an optimal solution using the algorithm of the simplex method.

16. The geometrical meaning of the simplex method.

17. The theoretical aspects of the simplex method.

18. A problem with mixed constraints.

19. The method of artificial basis.
20. The features of solving LPP which are given in the general form of LPP for solving economic problems.

### **Theme 3. Duality theory and analysis of linear models of economic optimization problems**

1. The basic concepts of duality theory.
2. Mutual duality problems of linear programming.
3. The economic meaning of the primal and dual LPP as an example of the product mix problem.
4. The rules of construction of a mathematical model of the dual problem.
5. The basic duality theorems and their economic explanation.
6. Finding an optimal solution to the initial problem using the dual problem.
7. Postoptimization analysis of LPP.
8. Dual estimations and the shortage of resources in the neighbourhood of the optimal solution to LPP.
9. The analysis of a range of changes of components of the matrix-column of the right part of the basic system of constraints.
10. The analysis of a range of changes of coefficients of the objective function.
11. The analysis of a range of changes of coefficients of the basic matrix of the constraints system.

### **Theme 4. The transportation problem**

1. Solving the transportation problem using the criterion of costs.
2. The statement of the transportation problem using the criterion of the transportation cost.
3. Finding the support basic solution.
4. Transformation to the other basic solution.
5. The problem of solution degeneracy of the transportation problem and ways to eliminate the degeneracy.
6. Finding the optimal solution using the method of potentials.
7. The criterion of the optimality of the solution.
8. The method of potentials.
9. The economic meaning of potentials.
10. Investigation of stability of an optimal solution as a problem of parametric programming.



11. Transportation problems with additional conditions.
12. Solving a transportation problem using the criterion of time.
13. Problems with economic content which are reduced to transportation problems.

### **Theme 5. Integer programming**

1. The economic statement of the integer programming problem.
2. A mathematical model of the integer programming problem.
3. An assignment problem.
4. Solving an assignment problem as a transportation problem.
5. The investment portfolio as a problem of combinatorial optimization.
6. The geometrical explanation of solutions to the integer programming problem on a plane.
7. Cutting methods.
8. Combinatorial methods.
9. Methods of approximate computations.
10. The branch and bound method.
11. Gomory method (the cutting method).
12. Generation of additional constraints.
13. Examples of economic problems which require using models of integer programming.

### **Theme 6. Nonlinear optimization models of economic systems**

1. Economic and mathematical statements of a problem of nonlinear programming.
2. The geometrical meaning (interpreting) of a problem of nonlinear programming.
3. The basic difficulties which arise in solving problems of nonlinear programming.
4. The necessary and sufficient conditions of the existence of a saddle point. Kuhn – Tucker theorem.
5. The economic statement and mathematical models of some quadratic programming.
6. Problems of convex programming and methods for solving them.
7. The economic and a mathematical statement of a linear fractional programming problem.

8. The geometrical meaning of a linear fractional programming problem.
9. Solving a linear fractional programming problem with the help of reducing it to a linear programming problem.

**Theme 7. Game theory. Analysis and risk management in economics on the basis of the concept of game theory**

1. The basic notions of game theory.
2. A mathematical model of the matrix game as a particular case of probabilistic models of economic systems.
3. Matrix games of two players.
4. The payoff matrix.
5. The minimax criterion.
6. The maximin criterion.
7. The game price.
8. The game in pure strategies.
9. A saddle point.
10. The mixed strategies game.
11. The basic theorem of game theory (Neumann theorem).
12. Reducing a matrix game of two players to a linear programming problem.
13. A geometrical explanation of the matrix game of two players.
14. Finding active strategies of players.
15. The content of the basic ways of quantitative pricing risk.
16. The system of quantitative estimations of the measure of an economic risk.
17. A game price as the risk appraisal.

**Thematic module 2  
Econometric methods**

**Theme 8. Particular properties of construction of econometric models and ways of construction**

1. Particular properties of econometric models.
2. The role and place of econometric models in the analysis of socio-economic systems.
3. An economic model.
4. The problems of econometric modelling.

5. Forming of a set of observations.
6. The concept of homogeneity of observations.
7. The accuracy of initial data.
8. The basic steps of construction of an econometric model.
9. The general characteristic of the basic steps of construction of an econometric model.
10. Particular properties of interpreting of the form of an econometric model.
11. Checking the statistical significance of a model.
12. Characteristics and criteria of the quality of econometric models.
13. Statistical estimations of parameters of econometric models.

### **Theme 9. A pair linear model**

1. The specification of a model.
2. Linear regression and correlation: the content and the estimator of parameters.
3. Estimation of parameters of a linear model of pair regression with the help of the method of least squares (MLS).
4. Checking the quality of the constructed pair linear model.
5. The estimator of the statistical significance of coefficients of regression and correlation.
6. Nonlinear regression.
7. Variance analysis.
8. The determination coefficient.
9. Checking the adequacy of an econometric model.

### **Theme 10. Methods of construction of a multiple regression model**

1. The general questions of a construction of a multiple regression model.
2. The specification of a model.
3. Estimation of parameters a regression equation.
4. Methods of construction of a general linear regression.
5. The method of least squares (MLS).
6. Statistical properties of MLS estimators.
7. The variance and standard errors of parameters of equations.
8. Estimation of parameters of a linear equation of multiple regression.
9. Interval estimators of coefficients of a theoretical regression equation.

10. Analysis of the quality of an empirical equation of multiple linear regression.
11. A partial equation of regression.
12. Multiple correlation.
13. Partial correlation.
14. Checking the statistical significance of coefficients of a regression equation.
15. Checking the general quality of a regression equation.
16. Prediction according to regression models.
17. The accuracy of prediction.

**Theme 11. Problems in the construction of linear multiple regression models**

1. Different aspects of multiple regression and problems which arise as a result of disorder of conditions of using MLS.
2. Multicollinearity.
3. The sequences of multicollinearity.
4. Methods of elimination of multicollinearity.
5. Heteroscedasticity.
6. Methods of defining heteroscedasticity.
7. The generalized method of least squares.
8. Autocorrelation of the model of residuals.
9. Methods of an elimination of autocorrelation of the model of residuals.
10. Implications of autocorrelation of the model of residuals.
11. The Durbin – Watson test.
12. Estimation of parameters of a model with autoregression.
13. Interpreting problems of parameters of a multifactor model.

**Theme 12. The generalized schemes of regression analysis**

1. The generalized schemes of regression analysis.
2. The generalized Aitken method of least squares.
3. Dummy variables.
4. Advanced methods of regression analysis.

**Theme 13. The systems of econometric equations**

1. The general concept of a system of equations which are used in econometrics.
2. The structure and reduced forms of a model.

3. An identification problem.
4. Estimation of parameters of a structural model.
5. The two-sweep method of least squares (2SMLS).
6. Economic models on the basis of the system of structural equations.

### **Theme 14. Dynamic econometric models**

1. Econometric models with lag variables.
2. The concept of a lag.
3. The concept of a lag variable.
4. The general characteristics of models with separated lags.
5. The types of lag models.
6. Interpreting the parameters of models with a separated lag.
7. Defining the lag structure.
8. Lags of independent variables.
9. A mutual correlation function.
10. A correlogram.
11. The choice of a model type with separated lags.
12. Methods of partitioning of dynamic econometric models.
13. Almon's method.
14. Koyck's method.

## **7.4. The independent test**

### **7.4.1. The basic requirements for carrying out the independent test**

The purpose of carrying out an independent test is the formation of students' practical skills in the use of theoretical knowledge of the academic discipline "Economic and Mathematical Methods" for solving economic problems and optimal decision making, obtaining skills in economic mathematical analysis and modelling for finding and explanation of the most effective solutions, as well as using the methods of quantitative and qualitative analysis of applied economic mathematical models.

The independent test should be carried out on the scheduled date. Besides, the description of each of the tasks for the independent test should be done (except the didactic analysis and the definition of corresponding elements of the independent work) according to the general technology of fulfillment:

learning and citing the basic questions of the theoretical material out of the recommended sources;

the design of the report on carrying out the task for the independent test, answers to control questions;

handing in the fulfilled tasks of the independent test and the answers to the control questions to the lecturer.

The fulfillment of the tasks of the independent test on the academic discipline is assessed depending on:

the understanding, the degree of mastering the theory and methodology of the problems which are considered;

the degree of acquaintance with the recommended literature and the mastery of the factual material of the academic discipline;

the ability to connect theory and practice in the consideration of practical situations, solving problems, carrying out calculations, fulfillment of tasks given for independent work;

the completeness of taking into account the conditions for the fulfillment of the tasks;

the logic of the given material and correspondence of its structure to the provided thematic elements of the task; the availability and completeness of consideration of the key concepts (definitions, terms, varieties and so on) of the subject matter of the task; the availability and explanations of the student's final conclusions; illustration of the processed material with the help of student's own examples and graphical material.

## **7.5. Preparation of the independent creative work**

Independent creative work of students is an integral part of the educational process. It forms the skills in the creation of major types of work (term papers, a diploma project). That is the reason why it is necessary for students to learn how to qualitatively prepare a creative work.

Within the framework of the given form of student's independent work it is proposed to prepare a presentation on the theme formulated for a student, in the electronic form (with the help of MS PowerPoint). An alternative may be a presentation at a next lecture or writing a scientific article.

Preparation of independent creative work provides for systematization, consolidation, broadening of the theoretical and practical knowledge of the academic discipline and using it in the process of solving a specific economic problem, development of skills in independent work and mastering the methods of investigation and experiment connected with the theme of the independent creative work.

Independent creative work stipulates for the availability of the following elements of scientific investigation: practical significance, a comprehensive systematic approach to solving the tasks of the investigation, the theoretical use of the progressive modern methodology and scientific developments, availability of the elements of creativity, the ability to use modern technologies.

A comprehensive systematic approach to the development of the theme of the independent creative work implies consideration of the subject of the research from different points of view that is from the position of a theoretical basis and practical ground work, conditions of its realization, analysis, explanations of ways for improvement in close relationship and a common logic of exposition.

The use of modern technology consists in the fact that in the process of fulfillment of analysis and explanation of ways for improvement of particular aspects of the subject and the object of investigation, a student has to use information about high achievements in techniques and technologies of investigation, use varied mathematical methods and ways, approaches to the definition and explanation of indicators of analysis of a social economic system or its elements.

Students submit the independent creative work to the lecturer in the electronic form if it is a presentation or in the printed or electronic form if it is a scientific publication.

After the complex presentation or a scientific publication has been reviewed and corrected by the lecturer, students make their presentations in front of the audience, report on the results stated in the scientific publication, make reports at a student's scientific and practical conference and so on.

## **8. Individual consultative work**

Individual consultative work is fulfilled according to the schedule of the individual consultative work in the following forms: individual studies, consultations, check of fulfillment of individual tasks, check and defence of the tasks presented for the current control and so on.

The forms of the individual consultative work are:

a) according to the mastery of the theoretical material:

consultations: individual (question-answer);

group (consideration of typical examples);

b) for complex assessment of the mastery of the program material: individual handing in of the fulfilled works.

## 9. Methods of study

To intensify the process of teaching the academic discipline "Economic and Mathematical Methods" the following educational technologies are applied: problem lectures, minilectures, work in small groups, discussions, brainstorming, moderations, presentations, computer simulation (games), the Delphi method, the method of scenarios, banks of visual support (Tables 9.1 and 9.2).

The basic difference of active and interactive methods of education from traditional ones is not only defined by the methods and techniques of teaching, but also by high effectivity of the educational process, which reveals itself in the high motivation of students; consolidation of theoretical knowledge in practice; raised students' consciousness; forming the ability to make independent decisions; forming the ability to approve collective decisions; forming the ability for social integration; getting skills in resolving conflicts; development of the ability to reach compromises.

**Problem lectures** are directed at the development of students' logical thinking. The theme is confined to two or three key issues, students' attention is concentrated on the material which has not been represented in textbooks, the experience of foreign educational universities is used with handing out printed materials to students during the lecture and drawing basic conclusions as to the issues considered. In the course of lectures students are asked questions for independent reflection which a lecturer answers himself, without waiting for students' answers. This kind of system makes students concentrate and begin to actively think in search of a correct answer.

**Minilectures** provide for the delivery of the educational material during a short-length segment of time and they are characterized by a significant content, complexity of logical constructions, forms, proofs and generalizations. They are conducted, as a rule, as a part of a study-investigation. Minilectures differ from full-size lectures by a shorter duration. Usually, they last no more than 10 – 15 minutes and they are used to briefly give new information to all students. Minilectures are often used as parts of a whole theme, which it is desirable to teach as a full-size lecture in order to avoid the audience's getting tired. Then the information is given by turn as several particular fragments, with other forms and methods of study used between them.

**Seminar-discussions** provide for exchange of thoughts and ideas of students on the given theme and develop thinking, help to form ideas and beliefs, produce skills in formulating thoughts and expressing them, teach to assess other people's proposals, critically come to personal ideas.



**Work in small groups** gives an opportunity to structure practical studies in the form and content, gives a possibility for each student's partaking in the work on the theme under study, stimulates forming personal qualities and experience of social communication.

Table 9.1

**Distribution of forms and methods of intensification of the educational process according to the themes of the academic discipline**

Theme	Practical application of educational technologies
1	2
<b>Thematic module 1 Optimization methods</b>	
<i>Theme 1.</i> The general theoretical foundations of optimization methods and models in economics	A problem lecture on the theme: "The choice of a method of solving economic and mathematical problems and explanation of this choice"
<i>Theme 2.</i> Problems of linear programming and methods for solving them	A minilecture on the theme: "A set of feasible solutions and an optimal solution to LPP". A problem lecture on the theme: "The method of artificial basis"
<i>Theme 3.</i> Duality theory and analysis of linear models of economic optimization problems	A problem lecture on the theme: "The analysis of a range of changes of components' coefficients in the construction of dual problems". Work in small groups with further discussion of the results of laboratory work
<i>Theme 4.</i> The transportation problem	A minilecture on the theme: "Finding the support basic solution to a transportation problem"
<i>Theme 5.</i> Integer programming	A minilecture on the theme: "An assignment problem" A minilecture on the theme: "The investment portfolio as a problem of a combinatorial optimization". Work in small groups with further discussion of the results of laboratory work
<i>Theme 6.</i> Nonlinear optimization models of economic systems	A problem lecture on the theme: "The statement and the geometrical meaning of a nonlinear programming problem"
<i>Theme 7.</i> Game theory. Analysis and risk management in economics on the basis of the concept of game theory	A minilecture on the theme: "A game price as the risk appraisal". Work in small groups with further discussion of the results of laboratory work. Presentation of independent creative work

Table 9.1 (the end)

1	2
<b>Thematic module 2 Econometric methods</b>	
<i>Theme 8.</i> Particular properties of construction of econometric models and ways of construction	A minilecture on the theme: "Checking the statistical significance and the quality of a model". Work in small groups with further discussion of the results of laboratory work
<i>Theme 9.</i> A pair linear model	A problem lecture on the theme: "Estimation of parameters of a linear model of pair regression with the help of the method of least squares"
<i>Theme 10.</i> Methods of construction of a multiple regression model	A minilecture on the theme: "The general questions of construction of a multiple regression model". Work in small groups with discussion of the results of laboratory work
<i>Theme 11.</i> Problems in the construction of linear multiple regression models	A problem lecture on the theme: "Different aspects of multiple regression and problems which arise as a result of disorder of conditions of using MLS"
<i>Theme 12.</i> The generalized schemes of regression analysis	A minilecture on the theme: "The generalized Aitken method of least squares"
<i>Theme 13.</i> The systems of econometric equations	A problem lecture on the theme: "The two-stage method of least squares of solving systems of econometric equations"
<i>Theme 14.</i> Dynamic econometric models	A problem lecture on the theme: "Methods of partitioning of dynamic econometric models. Almon's method. Koyck's method"

**Brainstorming** is a method of solving urgent tasks, its core lies in expressing as many ideas as possible in a short period of time, discussing and selecting them.

**Presentations** are speeches to students which are used for presenting certain achievements, group work results, reports of individual task fulfillment, instruction, demonstration of new goods and services.

**The method of scenarios** is used for designing probabilistic models of behavior and development of concrete events in the long term.

### Using the methodologies of intensification of the educational process

The theme of the academic discipline	Practical application of educational technologies	The methodologies of intensification of the educational process
1	2	3
<b>Thematic module 1</b> <b>Optimization methods</b>		
<i>Theme 1.</i> The general theoretical foundations of optimization methods and models in economics	<i>Practical study. Laboratory work.</i> Task: using built-in functions of MS Excel. Possibilities of realization of search for a solution to optimization problems with the help of the package of applied programs for personal computer	Work in small groups, moderation. Computer simulation
<i>Theme 2.</i> Problems of linear programming and methods for solving them	<i>Practical study. Laboratory work.</i> Task: solving linear programming problems with the help of the simplex method based on the example of product mix problems and diet problems	Work in small groups with discussion of the results of laboratory work. Computer simulation
<i>Theme 3.</i> Duality theory and analysis of linear models of economic optimization problems	<i>Practical study. Laboratory work.</i> Task: postoptimization analysis of LPP. Dual estimations and the shortage of resources in the neighbourhood of the optimal solution to LPP	Computer simulation. Work in small groups, the Delphi method
<i>Theme 4.</i> The transportation problem	<i>Practical study. Laboratory work.</i> Task: investigation of stability of an optimal solution relative to supplies and demands, costs and transportation of a unit of goods	A brainstorm, discussion, moderation
<i>Theme 5.</i> Integer programming	<i>Practical study. Laboratory work.</i> Task: solving integer programming problems	Work in small groups, brainstorms, the Delphi method
<i>Theme 6.</i> Nonlinear optimization models of economic systems	<i>Practical study. Laboratory work.</i> Task: examples of economic problems which require using models of integer programming	A brainstorm. The method of scenarios

Table 9.2 (the end)

1	2	3
<i>Theme 7.</i> Game theory. Analysis and risk management in economics on the basis of the concept of game theory	<i>Practical study. Laboratory work.</i> Task: forming an investment portfolio as a quadratic programming problem; basic ways of quantitative pricing risk	Work in small groups, a discussion, brainstorming, moderation
<b>Thematic module 2 Econometric methods</b>		
<i>Theme 8.</i> Particular properties of construction of econometric models and ways of construction	<i>Practical study. Laboratory work.</i> Task: construction of a linear pair regression model; checking the statistical significance of a model; characteristics and criteria of the quality of econometric models	Work in small groups, brainstorming, computer simulation, situational analysis
<i>Theme 9.</i> A pair linear model	<i>Practical study. Laboratory work.</i> Task: checking the quality and statistical significance of a constructed pair linear model	Work in small groups, brainstorming, computer simulation, presentations, discussion
<i>Theme 10.</i> Methods of construction of a multiple regression model	<i>Practical study. Laboratory work.</i> Task: construction of a multifactor linear model and checking its statistical significance and general quality	Work in small groups, brainstorming, computer simulation, discussion of the theoretical material
<i>Theme 11.</i> Problems in the construction of linear multiple regression models	<i>Practical study. Laboratory work.</i> Task: investigation of problems of a linear multifactor model; defining statistical estimations of parameters of a multifactorial econometric model	Work in small groups, brainstorming, computer simulation
<i>Theme 12.</i> The generalized schemes of regression analysis	<i>Practical study. Laboratory work.</i> Task: confidence limits for a regression line; confidence limits in a multiple regression	Work in small groups, brainstorming, computer simulation, presentations, discussion
<i>Theme 13.</i> The systems of econometric equations	<i>Practical study. Laboratory work.</i> Task: construction of economic models on the basis of the system of structural equations	Work in small groups, brainstorming, computer simulation, presentations, discussion
<i>Theme 14.</i> Dynamic econometric models	<i>Practical study. Laboratory work.</i> Task: construction of dynamic econometric models	Work in small groups, brainstorming, computer simulation

**Moderation** is a way to conduct a discussion, which leads quickly to concrete results, gives a possibility for all present students to take part in the process of search for a solution to a problem and take full responsibility for the result. The function of the moderator is to see to it that the rules of the discussion are observed, which gives a possibility to simplify the process of the search for a solution without interfering in its essence.

**The Delphi method** is used for the purpose of reaching a consensus in expert judgements. It gives a possibility for students to express their thoughts to a group of experts, which work individually in different places. To choose a management decision according to this method, the academic group is divided, for example, into five small groups. Four groups work, develop and make a management decision, and the fifth group is the expert team. This group carries out analysis of the variants of management decisions, which are proposed by the working groups, and assesses these variants. Within the expert group the distribution of its members according to specializations is fulfilled.

**A business game** is a method of imitation of making administrative decisions in various situations by means of playing according to the rules which have been worked out or are worked out by the members themselves. This method is realized through students' independent solving the set problem provided a shortage of the necessary knowledge when students themselves are forced to master the new content or search new connections in the learnt material.

**Computer simulation (game)** is an education method, which is based on the use of a specific computer program in order to get visual modelling of a process. Students can change the parameters and data, decisions and analyze the results of such decisions. The purpose of using this method is the development of systematic thinking of students, their ability to plan, form skills in identifying and analyzing problems, compare and estimate alternatives, make optimal decisions and work under the conditions of limited time.

**Banks of visual support** help to intensify the education process of studying the themes of the academic discipline with the help of visualization.

**The interactive distant education** is a set of educational technologies based on the principles of contact in the information educational space. They serve to organize the education of users distributed in the space and time.

## 10. Methods of control

The system of assessment of competences which were formulated for a student during the learning of the academic discipline (Table 2.1) takes into consideration the forms of studies which according to the syllabus of the academic discipline provide lectures, practical studies, laboratory work, fulfillment of students' independent work. The assessment of the formed competences of students is carried out on the accumulative 100-point system. According to the temporary provision "About the Order of Assessment of Students' Academic Performance on the Accumulative Point Rating System" of Simon Kuznets Kharkiv National University of Economics control ways include:

**current control** which is carried out within the term during lectures, practical studies and laboratory work and is assessed as a sum of accumulative points (the maximum equals 60 points; the minimum which makes it possible for a student to pass an exam, equals 35 points);

**module control** which is carried out in the form of a colloquium with taking into account the current control according to a corresponding thematic module, provides an integral assessment of student's results after learning the material of a logically completed part of the discipline (or a thematic module);

**final/term control**, which is carried out as a terminal exam, according to the schedule of the educational process.

*Current control* on the given academic discipline is carried out in the following forms:

- active in-class work (lecture);
- active in-class work (practical study);
- active in-class work (laboratory study);
- homework;
- competence-oriented tasks (defence of laboratory work);
- an independent test;
- a written test;
- independent creative work.

**A colloquium** is a form of reviewing and assessment of students' knowledge in the system of institutes of higher education. *The purpose* of carrying out a colloquium is to clarify the theoretical and practical knowledge obtained by a student as a result of listening to lectures, attendance of practical and laboratory studies and independent learning of the material. Within the bounds of the assigned purpose, the following *tasks* are fulfilled: evaluation of the quality and degree of student's understanding of the lecture

material; the development and fixing of the skills in expressing thoughts; the development of student's ability for independent single-minded preparation; the development of skills in the generalization of different literary sources; giving a possibility for a student to compare different points of view on a given question. A colloquium is conducted as an intermediate miniexam on the initiative of the lecturer and includes theoretical questions and practical tasks on the academic discipline. The list of questions, which are included into a colloquium on the themes of the thematic module, contains questions for self-assessment.

*Final/term control* is conducted in the form of a term exam. **Term exams** are a form of assessment of students' final mastery of the theoretical and practical material of a particular module of the academic discipline or the academic discipline on the whole, which are conducted as tests.

**The order of conducting the current assessment of students' knowledge.** Assessment of student's knowledge during practical studies and carrying out laboratory work is conducted on the accumulative system according to the following criteria:

- understanding, the degree of the mastery of the theory and methodology of the problems which are considered;

- the degree of the mastery of the factual material of the academic discipline;

- familiarizing with the recommended literary sources and modern literature on the questions which are considered;

- the ability to connect theory and practice in the consideration of particular examples, solving problems, carrying out laboratory work, carrying out calculations in the process of doing homework and tasks which are considered in class;

- the logic, structure, style of presenting the material in written papers and oral answers in class, the ability to ground one's own position, carry out generalization of the information and draw conclusions.

The maximum possible points which correspond to a particular task, are given on the condition of the correspondence of the solved task or the oral answer of a student to all the defined criteria. Lack of one or another component decreases the number of accumulative points. In the assessment of tasks set for independent work in laboratory and practical studies, the quality of fulfillment is also considered. Besides, handing in the performed task to the lecturer in accordance with the period defined by the schedule of the educational process plays an important role. If one of these conditions is not satisfied, the points are decreased.

A written test is carried out 2 times during a term and it includes practical tasks of different level of difficulty (complexity) according to the themes of the thematic module.

**The criteria for assessment of the written test are as follows:**

**5 points** if the test has been carried out without mistakes and deficiencies, all the tasks contain the necessary explanations, illustrations, analysis of the results and conclusions;

**4 points** if the test has been carried out, but there are no more than one mistake and no more than one deficiency or no more than three deficiencies;

**3 points** if no less than 2/3 of the test have been carried out, there are no more than two mistakes and no more than two deficiencies;

**2 points** if less than 2/3 of the test have been performed and the number of mistakes and deficiencies exceeds the norm for the mark of three points;

**1 point** if the fulfillment of the tasks has not been begun, but there is a particular correct thinking;

**0 point** if the task is unavailable.

*Revision and marking of the competence-oriented tasks* (defence of laboratory work on the themes which are combined into a corresponding thematic module) is carried out twice during a term in the form of work in small groups. Besides, the quality of fulfillment of the tasks for laboratory work and the ability to present the results of investigations, give reasonable answers to the questions of opponents, think critically, assess the results of the work of other participants must be assessed.

A colloquium is carried out twice during a term in the written form or in the form of an oral test for controlling students' knowledge of the theoretical material and the mastery of the categorical apparatus.

**The criteria for assessment of a colloquium:**

**6 points** if deep knowledge of the syllabus material has been demonstrated, a sequential, complete and logical answer has been given, a correct decision has been made, the mastery of different methods and techniques in carrying out practical tasks has been demonstrated;

**5 points** if knowledge of the syllabus material has been demonstrated, an answer without essential inaccuracies has been given, mastery of the necessary methods in carrying out practical tasks has been demonstrated;

**4 points** if knowledge of the basic material has been demonstrated, an answer with inaccuracies has been given, mastery of the necessary methods in carrying out practical tasks has been demonstrated;



**3 points** if knowledge of the basic material has been demonstrated, an answer with inaccuracies and quite incorrect formulations has been given, mistakes have been made in the use of the necessary methods in carrying out practical tasks;

**2 points** if knowledge of the basic material has not been demonstrated, an answer with essential mistakes and incorrect formulations has been given, lack of skills in the use of the necessary methods in carrying out practical tasks has been demonstrated;

**1 point** if an incorrect solution has been given, the fulfillment of the colloquium practical tasks has not been begun, but some particular correct thinking has been shown;

**0 point** if the task is unavailable.

**The criteria for assessment of independent work of students.** The general criteria for the assessment of independent work of students are profound and deep knowledge, the level of thinking, skills in the systematization of knowledge on particular themes, skills in drawing conclusions, attainments and techniques of carrying out practical tasks, the ability to find necessary information, carry out its classification and processing, self-realization in practical and laboratory studies.

The criteria for assessment of independent creative work and independent tests are:

the ability to carry out a critical independent estimation of the defined problems;

skills in the explanation of alternative views and availability of a students' own point of view, position on the defined problem;

using the analytical approach; the quality and accuracy of expressing the thought;

the logic, structure and explanation of conclusions about a particular problem;

independence of carrying out the work;

grammatical correctness of the presentation of the material; using the methods of comparison, generalization of the concepts and facts;

the design of the work;

the quality of presentation.

**The order of final control on the academic discipline.** The final control of knowledge and competences of students on the academic discipline is carried

out on the basis of the term exam. The examination paper includes the syllabus of the discipline and provides for assessment of the knowledge level and degree of the mastery of corresponding competences of students (Table 2.1).

The purpose of the exam is to test student's understanding of the syllabus material on the whole, the logic and relations between its particular parts, the skills in the creative use of the gained knowledge, the ability to formulate their attitude to a particular problem of the academic discipline and so on. The competent approach to the assessment of the exam implies measuring the level of the student's mastery of the competences provided by the qualifying requirements.

Each examination paper contains 5 practical tasks, including two first-level (diagnostic) tasks, two second-level (situational) tasks and one third-level (diagnostic and heuristic) task.

The structure of the examination paper is given in Table 10.1.

Table 10.1

### The structure of the examination paper

Task level	The content of tasks according to the themes
1	2
First	<p>Defining the lower and upper game prices, the existence of a saddle point. If there is no saddle point, solving a matrix game with the help of a graphical method in mixed strategies, i.e. conducting graphical analysis, getting optimal strategies of two players. Finding their probabilities and defining the game price.</p> <p>Checking the balance of the model of a transportation problem (defining openness or closedness of the problem model).</p> <p>If there is a closed model, obtaining an initial solution with the help of the minimal cost method or the north-west corner method.</p> <p>Checking the condition of a number of occupied cells (checking nonsingularity or singularity of a model). Then checking optimality of the initial solution with the help of the method of potentials.</p> <p>Construction of a cycle and performing redistribution of goods and a new solution to the problem</p>
Second	<p>Using the basic notions of correlation and regression analysis for construction of a model of linear regression in applied economic problems; defining errors of the estimation of parameters of the regression equation and construction of a confidence interval of the regression; checking the significance of the regression equation and regression coefficients with the help of Fisher and Student's t-test criteria</p>

Table 10.1 (the end)

1	2
	Using the basic notions of multiple correlation and regression for construction of linear multiple regression model of applied economic problems: construction of a multiple standardized equation of linear regression, construction of a multiple equation of linear regression in natural form; calculation of multiple coefficients of regression and determination; checking the significance of the regression equation and regression coefficients with the help of Fisher and Student's t-test criteria
Third	Solving a linear programming problem. Construction of economic and mathematical models using the given equation. Plotting a graphical drawing of each constraint of the constraints' system and nonnegativity conditions. Defining a solution polygon, finding the normal vector and the level line, the direction of movement of the level line, obtaining an optimal solution and an optimal value, construction of dual problems using duality theorems. Conducting economic analysis of the obtained results

The examination paper was formed according to the form No. H-5.05, "About the Statement of Forms of Documents for Personnel Training in Higher Educational Establishments of the 1st – 4th Levels of Accreditation" which was approved by the Ministry of Education and Science. A sample examination paper is given below.

The assessment of the exam is carried out according to the temporary provision "About the Order of Assessment of Students' Academic Performance on the Accumulative Point Rating System" of Simon Kuznets Kharkiv National University of Economics.

In the case of irreproachable fulfillment of all the examination tasks with the demonstration of deep knowledge of the academic discipline, skills in the practical use of the formed competences which are based on the ability to analyze and solve a wide range of tasks, a high level of completing the written work the student obtains 40 points.

The assessment of a particular task is carried out according to the following criteria.

***The task of the first level is assessed as follows:***

**7 points** in the case of the exact use of the scientific terminology and symbols in the necessary logical sequence; a creative approach to solving original problems which require a high level of knowledge;

**6 points** in the case of the exact use of the mathematical terminology and symbols; irreproachable mastery of mathematical tools; correct use of mathematical methods, facts, formulas and relations for solving the task of the third level;

**5 points** if a logically right sequence of steps of solution has been chosen; all the key moments of the solution have been grounded; 1 – 2 slight mistakes or slips are possible in the calculations which don't influence the correctness of the further solution;

**4 points** if mastery of a small part of obligatory skills and attainments required by the syllabus of the academic discipline has been demonstrated in solving the tasks; the conclusions, reproduction of the syllabus material of the discipline has not always been shown;

**3 points** if the task has been solved only partially with initial right considerations, but there are mistakes which considerably influenced the process of the right solution of the task;

**2 points** if the task fulfillment has been begun, there are separate correct considerations, but a logical mistake has been made which resulted in an incorrect solution.

**1 point** if the condition has been written;

**0 point** if no task has been fulfilled.

***The task of the second level is assessed as follows:***

**8 points** if solving the assigned tasks is characterized by a creative use of the theoretical instrument, logical correctness, precision, explanation of conclusions, rationality or using original approaches to solving the tasks;

**7 points** if perfect mastery of the skill in the use of mathematical tools with application of information from other educational courses and disciplines has been demonstrated; one slight mistake has been made; a high level of standards of carrying out the tasks has been shown;

**6 points** if a logically right sequence of steps of solution has been chosen; all the key points of solution have been grounded; 1 – 2 slight mistakes or slips are possible in the calculations which don't influence the correctness of the further solution;

**5 points** in the case of the correct use of the terminology of the discipline and the basic methods for solving standard problems; showing the ability to use theoretical knowledge for solving standard (multistep) problems, some mistakes or deficiencies on the calculation stage of presentation of the solution; the ability to conclude;

**4 points** in the case of more than one mistake and one or two deficiencies in the calculations, graphs, the choice of the method of solution, which have caused a wrong final result in some cases;

**3 points** if the task fulfillment has been begun, there are separate correct considerations, but a logical mistake has been made which resulted in an incorrect solution;

**2 points** if numerical gross mistakes have been made in the process of using the concepts of the discipline in the formulas which prove the absence of a minimum necessary part of the compulsory skills and the practical attainments provided for the discipline syllabus;

**1 point** if no task fulfillment has been begun, but the condition has been written;

**0 point** if no task fulfillment has been begun.

***The task of the third level is assessed as follows:***

**10 points** if the ability for scientific investigative developments on the problems of the discipline has been shown; perfect skills in the use of mathematical tools and modern scientific theoretical approaches, a high level of standards of carrying out tasks have been demonstrated;

**9 points** in the case of using scientific terminology and symbols in the necessary logical sequence; solving the assigned tasks characterized by precision, explanation; a creative approach; rationality of the choice of the method of solution; correct necessary calculations and transformations;

**8 points** if systematic, deep and full knowledge of all the parts of the academic discipline and the basic questions which go beyond the discipline has been shown; a high level of standards of carrying out the tasks has been demonstrated;

**7 points** in the case of sporadic slight deficiencies which don't influence the final result; correct use of mathematical methods, facts, formulas and relations for solving the task of different level of complexity;

**6 points** if the ability to conclude and compare the theoretical and practical material has been demonstrated; correct (but not always rational) use of mathematical methods of solution, facts, formulas and relations has been shown;

**5 points** if half of the task has been done, the interpretation of the obtained results is unavailable; the level of the standards of carrying out the task is acceptable;

**4 points** if the tasks have been carried out without any logical relationship of the mathematical concepts; and practical solutions have not been given sufficient theoretical explanation;

**3 points** if an acceptable volume of knowledge has been shown within the educational standard; the use of mathematical symbols and terminology has been insufficient and inexact, the knowledge of the basic formulas and concepts on the discipline has not been demonstrated;

**2 points** in the case of solving the tasks with the theoretical material used only on the level of concepts; the inability to understand the connection of the theoretical material with the practical tasks;

**1 point** if the condition has been written;

**0 point** if no task fulfillment has been begun.

### A sample examination paper

Form No. H-5.05

## SIMON KUZNETS KHARKIV NATIONAL UNIVERSITY OF ECONOMICS

**Educational level: bachelor**

**Specialization: 292 "International Economic Relations"**

**Term 1**

**Academic discipline: "Economic and Mathematical Methods"**

### Examination paper

**Task 1.** The payoff matrix is given by:  $\Pi = \begin{pmatrix} 7 & 4 & 1 & 7 & -2 \\ 5 & 0 & 4 & -3 & 2 \end{pmatrix}$ .

Solve the matrix game by the graphical method, find the lower price and the upper price of the game, the optimal strategies and the game price. Make the economic analysis of the obtained values in the problem.

**Task 2.** Suppose that three supply sources  $A_1$ ,  $A_2$ ,  $A_3$  have amounts 40, 60, 100 per units of identical goods that must be shipped to four consumers

$B_1, B_2, B_3, B_4$  with respective demands 45, 35, 55, 65 per units for these

goods. The transportation cost matrix is given:  $C = \begin{pmatrix} 4 & 1 & 2 & 3 \\ 3 & 2 & 3 & 7 \\ 4 & 4 & 5 & 2 \end{pmatrix}$ .

Do the following:

a) obtain a feasible initial solution to the transportation problem by the northwest corner method or the minimal cost method in order to find a flow of the least cost ( $Z_{\min}$ ) that ships from supply sources to consumer destinations;

b) verify whether this feasible initial solution is optimal;

c) construct a cycle and perform a redistribution of goods over this cycle if the current solution is not optimal. Explain the results.

**Task 3.** Data:  $\sum x_i = 81.3, \sum x_i^2 = 865.63, \sum y_i = 96.8, \sum x_i y_i = 735, \sum y_i^2 = 1194, n = 100$ .

Do the following:

a) construct a pair linear theoretical equation of the regression:

$$\hat{y}_x = b_0 + b_1 x;$$

b) calculate the correlation coefficient  $r$  and explain the obtained results.

**Task 4.** Verify the significance of regression coefficients at the significance level  $\alpha = 0.01$  if  $\beta_1 = -0.14, \beta_2 = -0.41, S_{\beta_1} = 0.07, S_{\beta_2} = 0.23, n = 100$  are given.

**Task 5.** The company produces articles of two kinds  $A_1$  and  $A_2$ , using raw material of three types  $S_1, S_2$  and  $S_3$  whose respective supplies are 10, 16 and 12 per day.

The expense rates of each type of raw material per one article are 3, 1 and 5 for  $A_1$  and 2, 4 and 5 for  $A_2$  respectively.

The profit per unit output of product  $A_1$  is 12, the profit per unit output of product  $A_2$  is 9.

Do the following:

- a) construct a mathematical model of this problem if the objective is to find a product mix maximizing the profit;
- b) solve it by the graphical method;
- c) draw conclusions about the quantities of product of each type and the remains of raw materials;
- d) construct a dual problem and solve it using duality theorems;
- e) explain the obtained results.

Approved at the meeting of the Department of Higher Mathematics and Economic Mathematical Methods.

Protocol No. 1 of August 28, 2017.

The head of the department

L. Malyarets

The lecturer

Ie. Misiura

A student, who for a valid reason, attested documentally, hasn't had a possibility to take part in the forms of current control, that is, hasn't passed the thematic module, has the right to complete it during two weeks after coming back to studies according to the notice of the dean of the department subject to a given period.

**A student can't be allowed to take the exam** if the number of points obtained during the current and module control according to the thematic module during the term does not make 35. After the examination period the dean of the department gives a notice about sitting the failed exams. In the given period the student adds the required points.

The final mark on the academic discipline is calculated according to the points obtained during the exam and the points obtained during current control on the accumulative system.

**The student's progress is assessed** if the number of points obtained as a result of all forms of control equals or exceeds 60.

Accordingly, the minimum possible number of points in the current and module control during the term equals 35 and the minimum possible number of points obtained in the exam equals 25.



The result of the terminal exam is assessed in points (the maximum is 40 points, the minimum possible number is 25 points) and entered into a corresponding column of the *Examination Record List*.

The final mark on the academic discipline is calculated according to the points obtained during the exam and points obtained during the current control on the accumulative system.

The total result in points during the term is: *60 and more points mean passed; 59 and fewer points mean failed*; and this is entered into the *Examination Record List* on the academic discipline.

## 11. The distribution of points which students obtain

An example of a technological chart of accumulative rating points and the system of assessment of the level of formed professional competences which a student of the day-time form of studies has to get, is given in Table 11.1 according to the forms of study and methods of control which are used in teaching the academic discipline.

Table 11.1

### The system of assessment of the professional competences formed

Professional competences	Educational week	Hours	Forms of study	Assessment of the level of the formed competences			
				Forms of control	Maximal point		
1	2	3	4	5	6		
<b>Thematic module 1 Optimization methods</b>					<b>26.4</b>		
AOM	1	Class	2	<b>Lecture</b>	<i>Theme 1.</i> The general theoretical foundations of optimization methods and models in economics	Active class work	0.3
			2	<b>Practical study</b>	The general theoretical foundations of optimization methods and models in economics. Problems of linear programming and methods for solving them	Active class work	0.3
		IWS	4	<b>Preparation for studies</b>	Search, choice and looking through literary sources on the themes of the academic discipline. Learning the lecture material and preparation for practical studies	There is no control of independent work	–

Table 11.1 (continuation)

1	2	3	4		5	6	
AOM	2	Class	2	<b>Lecture</b>	<i>Theme 2.</i> Problems of linear programming and methods for solving them	Active class work	0.3
			2	<b>Laboratory study</b>	Built-in functions of MS Excel. Elements of linear algebra in MS Excel	Active class work	0.3
		IWS	4	<b>Preparation for studies</b>	Learning the lecture material and preparation for practical studies. Carrying out practical homework and the independent test. Search of the material for the independent creative task	Homework	0.2
	3	Class	2	<b>Lecture</b>	<i>Theme 3.</i> Duality theory and analysis of linear models of economic optimization problems	Active class work	0.3
			2	<b>Practical study</b>	Duality theory and analysis of linear models of economic optimization problems	Active class work	0.3
		IWS	4	<b>Preparation for studies</b>	Search, choice and looking through literary sources on the theme. Carrying out practical homework and the independent test	Homework	0.2
	4	Class	2	<b>Lecture</b>	<i>Theme 4.</i> The transportation problem	Active class work	0.3
			2	<b>Laboratory study</b>	The simplex method of solving problems of linear optimization	Active class work	0.3
		IWS	4	<b>Preparation for studies</b>	Learning the lecture material and preparation for practical studies. Carrying out practical homework and the independent test	Homework	0.2
	5	Class	2	<b>Lecture</b>	<i>Theme 5.</i> Integer programming	Active class work	0.3
			2	<b>Practical study</b>	The transportation problem. Integer programming	Active class work	0.3
		IWS	4	<b>Preparation for studies</b>	Learning the lecture material and preparation for practical studies. Carrying out practical homework and the independent test	Homework. Independent test	0.2 + 5
6	Class	2	<b>Lecture</b>	<i>Theme 6.</i> Nonlinear optimization models of economic systems	Active class work	0.3	
		2	<b>Laboratory study</b>	The transportation problem	Active class work. Competence-oriented task	0.3 + 5	
	IWS	6	<b>Preparation for studies</b>	Search, choice and looking through literary sources on the theme. Carrying out practical homework and the independent test	Homework	0.2	

Table 11.1 (continuation)

1	2	3	4		5	6	
AOM	The ability to solve problems of game theory and construct nonlinear optimization models of economic systems	Class	2	<b>Lecture</b>	<i>Theme 7.</i> Game theory. Analysis and risk management in economics on the basis of the concept of game theory	Active class work. Colloquium	0.3 + 6
			2	<b>Practical study</b>	Nonlinear optimization models of economic systems. Game theory. Analysis and risk management in economics on the basis of the concept of game theory	Active class work. Written test	0.3 + 5
		IWS	6	<b>Preparation for studies</b>	Learning the lecture material. Carrying out practical homework. Preparation for the colloquium. Preparation for a written test	Homework	0.2
<b>Thematic module 2 Econometric methods</b>						<b>33.6</b>	
AEM	The ability to construct econometric models and knowing how to construct them	Class	2	<b>Lecture</b>	<i>Theme 8.</i> Particular properties of construction of econometric models and ways of construction	Active class work	0.3
			2	<b>Laboratory study</b>	Forming an investment portfolio as a quadratic programming problem	Active class work	0.3
		IWS	4	<b>Preparation for studies</b>	Learning the lecture material. Carrying out the practical homework and the independent test. Preparation for the written test	Homework	0.2
		Class	2	<b>Lecture</b>	<i>Theme 9.</i> A pair linear model	Active class work	0.3
			2	<b>Practical study</b>	Particular properties of construction of econometric models and ways of construction. A pair linear model	Active class work	0.3
		IWS	4	<b>Preparation for studies</b>	Search, choice and looking through literary sources on the theme. Carrying out the practical homework and tasks of the independent test	Homework	0.2
	The ability to use methods of construction of a multiple regression model	Class	2	<b>Lecture</b>	<i>Theme 10.</i> Methods of construction of a multiple regression model	Active class work	0.3
			2	<b>Laboratory study</b>	A linear pair regression model. Checking the significance of parameters of a pair regression model	Active class work	0.3
		IWS	5	<b>Preparation for studies</b>	Search, choice and looking through literary sources on the theme. Carrying out the practical homework and tasks of the independent test	Homework	0.2
	Class	2	<b>Lecture</b>	<i>Theme 11.</i> Problems in the construction of linear multiple regression models	Active class work	0.3	
		2	<b>Practical study</b>	Methods of construction of a multiple regression model	Active class work	0.3	
IWS	4	<b>Preparation for studies</b>	Search, choice and looking through literary sources on the theme. Carrying out the practical homework	Homework	0.2		

Table 11.1 (continuation)

1	2	3	4		5	6		
AEM	The ability to use generalized schemes of regression analysis	12	Class	2	<b>Lecture</b>	<i>Theme 12.</i> The generalized schemes of regression analysis	Active class work	0.3
			Class	2	<b>Laboratory study</b>	A multifactor linear model	Active class work	0.3
		IWS	5	<b>Preparation for studies</b>	Learning the lecture material. Carrying out practical homework and the independent test	Homework	0.2	
	The ability to construct systems of econometric equations and a linear multiple regression model	13	Class	2	<b>Lecture</b>	<i>Theme 13.</i> The systems of econometric equations	Active class work	0.3
				Class	2	<b>Practical study</b>	Problems in the construction of a linear multiple regression model. The generalized schemes of regression analysis	Active class work
			IWS	5	<b>Preparation for studies</b>	Learning the lecture material, preparation for practical studies. Carrying out practical homework and tasks of the independent test. Independent creative work	Homework. Independent test	0.3 + 5
	The ability to construct dynamic econometric models and investigate problems of a linear multifactor model	14	Class	2	<b>Lecture</b>	<i>Theme 14.</i> Dynamic econometric models	Active class work. Colloquium	0.3 + 6
				Class	2	<b>Laboratory study</b>	Investigation of problems of a linear multifactor model	Active class work. Competence-oriented task
			IWS	5	<b>Preparation for studies</b>	Search, choice and looking through literary sources on the theme. Carrying out the practical homework and tasks of the independent test. Preparation for the written test and independent creative work	Homework	0.2
		15	Class	2	<b>Practical study</b>	Systems of econometric equations. Dynamic econometric models	Active class work. Written test	0.3 + 5
			IWS	5	<b>Preparation for studies</b>	Search, choice and looking through literary sources on the theme. Carrying out practical homework. Preparation for the colloquium and the presentation of the independent creative work	Homework	0.2
			16	Class	2	<b>Laboratory study</b>	Confidence limits for a regression line. Confidence limits in a multiple regression	Active class work. Creative task
IWS	3	<b>Preparation for studies</b>		Preparation for the defence of laboratory work	Homework	0.2		

Table 11.1 (the end)

1	2	3	4	5	6	
Examination period	Class	2	<b>Preexam consultation</b>	Solving practical tasks according to the themes which are included in the final control	<b>Total control</b>	40
		2	<b>Exam</b>	Carrying out the tasks of the examination paper		
	IWS	10	<b>Preparation for the exam</b>	Review of the material of thematic modules		
<b>Total sum of hours</b>		150	<b>Total maximal number of points for the discipline</b>		<b>100</b>	
including						
<i>class</i>		<b>60</b>	40 %	<i>current control</i>	<b>60</b>	
<i>independent work</i>		<b>90</b>	60 %	<i>total control</i>	<b>40</b>	

The distribution of points according to the themes of the thematic modules is given in Table 11.2.

The maximum number of points which a student can accumulate during a week according to the forms and methods of study and control is given in Table 11.3.

The final mark on the academic discipline is defined according to the temporary provision "About the Order of Assessment of Students' Academic Performance on the Accumulative Point Rating System" of Simon Kuznets Kharkiv National University of Economics (Table 11.4).

Marks according to this scale are entered in the Examination Record List, the individual educational plan of a student and other academic documents.

Table 11.2

### The distribution of points according to the themes

Current testing and independent work														Final test (exam)	Sum		
1														2	3		
Thematic module 1							Thematic module 2							40	100		
T1	T2	T3	T4	T5	T6	T7	T8	T9	T10	T11	T12	T13	T14				
0.6	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	1.8				
Written test							Written test										
5							5										
Independent test							Independent test										
5							5										

Table 11.2 (the end)

1		2	3
Competence-oriented task	Competence-oriented task		
5	5		
Colloquim	Colloquim		
6	6		
Independent creative work			
6			

Note. T1, T2, ..., T14 are themes of the thematic modules.

Table 11.3

### The distribution of points through weeks

Themes of the thematic module			Lectures	Practical study	Laboratory study	Homework	Competence-oriented task	Independent test	Written test	Independent creative work	Colloquim	Total
Thematic module 1 Optimization methods	Theme 1	week 1	0.3	0.3	-	-	-	-	-	-		0.6
	Theme 2	week 2	0.3	-	0.3	0.2	-	-	-	-		0.8
	Theme 3	week 3	0.3	0.3	-	0.2	-	-	-	-		0.8
	Theme 4	week 4	0.3	-	0.3	0.2	-	-				0.8
	Theme 5	week 5	0.3	0.3	-	0.2	-	5				5.8
	Theme 6	week 6	0.3	-	0.3	0.2	5	-				5.8
	Theme 7	week 7	0.3	0.3	-	0.2	-	-	5		6	11.8
Thematic module 2 Econometric methods	Theme 8	week 8	0.3	-	0.3	0.2	-	-				0.8
	Theme 9	week 9	0.3	0.3	-	0.2	-	-				0.8
	Theme 10	week 10	0.3	-	0.3	0.2	-					0.8
	Theme 11	week 11	0.3	0.3	-	0.2	-					0.8
	Theme 12	week 12	0.3	-	0.3	0.2	-					0.8
	Theme 13	week 13	0.3	0.3	-	0.2	-	5				5.8
	Theme 14	week 14	0.3	-	0.3	0.2	5				6	11.8
		week 15	-	0.3	-	0.2	-		5		-	5.5
week 16		-	-	0.3	0.2	-	-	-	6	-	6.5	
<b>Total</b>			<b>4.2</b>	<b>2.4</b>	<b>2.4</b>	<b>3.0</b>	<b>10</b>	<b>10</b>	<b>10</b>	<b>6</b>	<b>12</b>	<b>60</b>

### The scales of assessment: national and ECTS

Sum of points including all forms of study	Mark on the ECTS scale	Mark on the national scale	
		for an exam, a term paper, practice	for a test
90 – 100	A	excellent	passed
82 – 89	B	very good	
74 – 81	C	good	
64 – 73	D	satisfactory	
60 – 63	E		
35 – 59	FX	unsatisfactory	failed
1 – 34	F		

## 12. Recommended reading

### 12.1. Main

1. Єгоршин О. О. Математичне програмування : підручник / О. О. Єгоршин, Л. М. Малярець. – Харків : ВД «ІНЖЕК», 2006. – 438 с.

2. Єгоршин А. А. Лабораторний практикум з економетрики в Excel : навчально-практичний посібник / О. О. Єгоршин, Л. М. Малярець. – Харків : Вид. ХНЕУ, 2011. – 140 с.

3. Збірник вправ з навчальної дисципліни "Економіко-математичне моделювання" для студентів усіх галузей знань усіх форм навчання / уклад. Л. М. Малярець, Е. Ю. Железнякова, Л. О. Норік. – Харків : Вид. ХНЕУ, 2009. – 88 с.

4. Лабораторний практикум з навчальної дисципліни "Економіко-математичне моделювання" : начальніо-практичний посібник / Л. М. Малярець, П. М. Куликов, І. Л. Лебедева та ін. – Харків : Вид. ХНЕУ, 2009. – 136 с.

5. Малярець Л. М. Економіко-математичні методи та моделі : навчальний посібник / Л. М. Малярець, Е. Ю. Железнякова, Є. Ю. Місюра. – Харків : Вид. ХНЕУ ім. С. Кузнеця, 2011. – 320 с.

6. Малярець Л. М. Економіко-математичні методи та моделі : навчальний посібник / Л. М. Малярець. – Харків : Вид. ХНЕУ ім. С. Кузнеця, 2016. – 405 с.

7. Малярець Л. М. Экономико-математические методы и модели : учебное пособие для иностранных студентов / Л. М. Малярець. – Харьков : Изд. ХНЭУ, 2013. – 288 с.

8. Малярец Л. М. Эконометрика в примерах и задачах / Л. М. Малярец, Э. Ю. Железнякова, Л. А. Норик. – Харьков : Изд. ХНЭУ им. С. Кузнеця, 2014. – 268 с.

9. Методичні рекомендації до виконання контрольних робіт з навчальної дисципліни "Економіко-математичне моделювання" для студентів усіх напрямків підготовки заочної форми навчання / уклад. Л. М. Малярець, Е. Ю. Железнякова, І. Л. Лебедева, Л. О. Норик. – Харків : Вид. ХНЕУ, 2008. – 36 с.

## 12.2. Additional

10. Англо-русский словарь математических терминов / под ред. П. С. Александрова. – Москва : Мир, 1994. – 416 с.

11. Лук'яненко І. Г. Економетрика: Практикум з використанням комп'ютера / І. Г. Лук'яненко, Л. І. Краснікова. – Київ : Товариство "Знання", КОО, 1998. – 220 с.

12. Наконечний С. І. Економетрія : підручник / С. І. Наконечний, Т. О. Терещенко, Т. П. Романюк. – Вид. 3-тє., доп. та перероб. – Київ : КНЕУ, 2005. – 520 с.

13. Наконечний С. І. Математичне програмування : навч. посібник / С. І. Наконечний, С. С. Савіна. – Київ : КНЕУ, 2005. – 452 с.

14. Пономаренко В. С. Багатовимірний аналіз соціально-економічних систем : навчальний посібник / В. С. Пономаренко, Л. М. Малярець. – Харків : Вид. ХНЕУ, 2009. – 384 с.

## 12.3. Methodical support

15. Економіко-математичні методи : опорний конспект [Електронний ресурс]. – Режим доступу : <http://www.ikt.hneu.edu.ua/course/view.php?id=929>.

16. Методичні рекомендації до виконання практичних завдань з навчальної дисципліни "Економіко-математичні методи" [Електронний ресурс]. – Режим доступу : <http://www.ikt.hneu.edu.ua/course/view.php?id=929>.

17. Методичні рекомендації та завдання для виконання лабораторних робіт з навчальної дисципліни "Економіко-математичні методи" [Електронний ресурс]. – Режим доступу : <http://www.ikt.hneu.edu.ua/course/view.php?id=929>.



# Appendices

Appendix A

Table A.1

## The structure of components of professional competences formed on mastering the academic discipline "Economic Mathematical Methods" according to Ukraine's national scale of qualifications

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Competence formed within the theme	Minimal experience	Knowledge	Skills and abilities	Communication	Autonomy and responsibility
1	2	3	4	5	6
<b>Theme 1. The general theoretical foundations of optimization methods and models in economics</b>					
Forming skills to use the instrument of mathematical programming for solving economic problems	Mastery of the notions of the classical optimization theory	Attainment of the notion of the choice of a method for solving economic and mathematical problems and explanation of this choice	The ability to construct economic and mathematical models, define the objective of modelling, choose a method of solving economic and mathematical problems	Understanding the role and place of mathematical programming in modern scientific research and their significance in further solving professional problems	A student <b>must be able to</b> construct economic and mathematical models, choose a method for solving economic and mathematical problems and explain this choice
<b>Theme 2. Problems of linear programming and methods for solving them</b>					
The ability to form and solve linear programming problems	Mastery of the notions: a polygon of plans, an objective function, a system of constraints, support and optimal plans; the statement of a problem and basic optimization methods	Attainment of basic methods of finding a solution to a linear programming problem	The ability to solve problems with the help of a graphical method and a simplex method	Understanding the role and place of linear programming in modern scientific research and their significance in further solving professional problems	A student <b>must be able to</b> solve problems of linear programming

Table A.1 (continuation)

1	2	3	4	5	6
<b>Theme 3. Duality theory and analysis of linear models of economic optimization problems</b>					
The ability to use duality theorems of finding a solution to a dual problem	Mastery of the notions: a primal problem; a dual problem; the first duality theorem; the second duality theorem	Attainment of basic methods of finding a solution to a linear programming problem	The ability to use duality theorems and find a solution to one of the pairs of symmetric mutually dual problems by a known solution to the other problem	Understanding the role and place of linear programming in modern scientific research and their significance in further solving professional problems	A student <b>must be able to</b> 1) <b>form</b> a dual problem for the given primal problem; 2) <b>find</b> a solution to a dual problem knowing the solution to the primal problem with the help of duality theorems; 3) <b>give</b> economic interpretations of solutions to the primal and dual problems
<b>Theme 4. The transportation problem</b>					
The ability to analyze economic problems which use principles of solving a transportation problem	Ways of solving a transportation problem: forming a support plan and checking the economic problems by the method of potentials	Attainment of the basic method of finding a solution to a transportation problem	The ability to solve transportation problems with the help of the method of potentials	Understanding the role and place of the transportation problem in modern scientific research and their significance in further solving professional problems	A student <b>must be able to</b> solve and find an optimal solution to transportation problems
<b>Theme 5. Integer programming</b>					
Forming skills to solve economic problems using integer programming	Skills to solve a problem of integer programming with the help of the sectioning method and the method of Gomory	Attainment of basic methods of finding a solution to an integer programming problem	The ability to solve a problem of integer programming with the help of the sectioning method (the method of branches and bounds) and the method of Gomory	Understanding the role and place of integer programming in modern scientific research and their significance in further solving professional problems	A student <b>must be able to</b> solve problems of integer programming

Table A.1 (continuation)

1	2	3	4	5	6
<b>Theme 6. Nonlinear optimization models of economic systems</b>					
The ability to form and solve nonlinear programming problems, know principles of elimination of difficulties in solving nonlinear programming problems	Skills to use methods of quadratic programming, convex programming and fractional programming, the method of Lagrange's multipliers	Attainment of the basic methods of finding a solution to nonlinear programming problems	Knowledge of the principles of elimination of basic difficulties in solving nonlinear programming problems	Understanding the role and place of nonlinear programming in modern scientific research and their significance in further solving professional problems	A student <b>must be able to</b> solve practical problems of nonlinear programming using the method of Lagrange's multipliers, solve practical problems of nonlinear programming if an objective function is nonlinear
<b>Theme 7. Game theory. Analysis and risk management in economics on the basis of the concept of game theory</b>					
The ability to form, solve and analyze problems of game theory, analyze and manage a risk in economics on the basis of game theory	Attainment of the basic notions of game theory, a mathematical model of a matrix game as a particular case of probabilistic models of the economic system	Attainment of the basic methods of finding a solution to problems of game theory, a system of qualitative estimators of the measure of an economic risk	The ability to construct a mathematical model of the matrix game as a particular case of probabilistic models of economic systems and use ways of quantitative assessment of risk	Solving economic problems using the methods of game theory. Using basic principles for estimation of risk	A student <b>must form, solve and analyze</b> problems of game theory, model a risk in economics, model conflict situations, define an optimal strategy of development of a situation, qualitatively estimate risks in economics
<b>Theme 8. Particular properties of construction of econometric models and ways of construction</b>					
The ability to fulfill basic steps in the construction of an econometric model, check the statistical significance of a model and define statistical estimations of parameters of econometric models	The ability to form a space for description of real objects and processes of economics	Mastery of the notions: econometric models, application and problems of econometric modelling	Knowledge of the basic steps of construction of an econometric model. Knowledge of the problems of econometric modelling. Knowledge of particular properties of interpreting the form of an econometric model	Checking the statistical significance of a model. Defining statistical estimations of parameters of econometric models	A student <b>must</b> 1) know the technology of defining values in economics; 2) form sets of observations of economic processes; 3) check their homogeneity and accuracy of initial data

Table A.1 (continuation)

1	2	3	4	5	6
<b>Theme 9. A pair linear model</b>					
The ability to construct a pair regression equation and verify the quality of the pair linear regression and the statistical significance of coefficients of the regression equation	The ability to form a space for description of real objects and processes of economics	Knowledge of the method of least squares and conditions for using it for estimation of parameters of a linear model of pair regression. Carrying out variance analysis	Calculation of the basic coefficients: parameters of a pair linear model, the correlation coefficient and the determination coefficient. Checking the adequacy of the econometric model	Checking the quality of the constructed pair linear model. Carrying out estimation of the statistical significance of coefficients of regression and correlation	A student <b>must</b> 1) model pair dependences of the resulting factor on the factors in economics; 2) analyze the obtained results on the basis of interpreting the parameters of the model
<b>Theme 10. Methods of construction of a multiple regression model</b>					
The ability to construct a multiple regression model and verify the quality of the multiple linear regression and the statistical significance of coefficients of the regression equation	Attainment of skills in using the method of least squares for construction of classical econometric models	Knowledge of the basic principles of selection of factors for construction of an econometric model of multiple regression and principles of choosing the form of a regression equation	Defining parameters of a multiple regression equation with the help of the method of least squares. Constructing regression equations in natural and standardized variables	Calculation of the variance and standard errors of a multiple regression equation. Using the definite statistical criteria for checking the quality of an econometric model	A student <b>must</b> 1) model pair dependences of the resulting factors on several factors in economics; 2) analyze the obtained results on the basis of interpreting the parameters of the model; 3) carry out analysis of the quality of an empirical equation of multiple regression
<b>Theme 11. Problems in the construction of linear multiple regression models</b>					
The ability to define multicollinearity, heteroscedasticity, autocorrelation and use methods of elimination of these phenomena	Attainment of skills in using the method of least squares for construction of classical econometric models	Knowledge of the principles of linearization of variables. Using the generalized method of least squares. Knowledge of the problems of parameters of a multiple regression model in economics	Knowledge of the reasons for multicollinearity, heteroscedasticity, autocorrelation and using the elimination methods. Defining the presence of multicollinearity and eliminating it using the decrease of the model dimension	Defining the presence of multicollinearity with the help of a matrix of pair coefficients of correlation	A student <b>must</b> 1) carry out transformation to a linear model; 2) use the generalized method of least squares for construction of econometric models; 3) estimate parameters of an econometric model with autoregression

Table A.1 (the end)

1	2	3	4	5	6
<b>Theme 12. The generalized schemes of regression analysis</b>					
The ability to use dummy variables and advanced methods of regression analysis for solving real economic problems	Attainment of skills in using the method of least squares for construction of classical econometric models	Knowledge of particular properties of construction of models with qualitative variables (dummy variables)	Using the generalized Aitken method of least squares. Knowledge of the advanced methods of regression analysis. Solving modern problems with the help of econometric models	Solving real economic problems using dummy variables, the generalized Aitken method of least squares and the advanced methods of regression analysis	A student <b>must</b> 1) construct regression models with qualitative variables and models where quantitative and qualitative variables are given such as models of productive efficiency; 3) know reasons and sequences of problems of real econometric models
<b>Theme 13. The systems of econometric equations</b>					
The ability to form econometric models on the basis of the system of structural equations	Attainment of skills in using the method of least squares for construction of classical econometric models	Knowledge of particular properties of construction of systems of econometric equations, estimation of parameters of a structural model	Using the two-stage method of least squares. Knowledge of Kane models	Construction of economic models on the basis of the system of structural equations	A student <b>must</b> construct macroeconomic models of functioning of national economy of the Kane type
<b>Theme 14. Dynamic econometric models</b>					
The ability to form econometric models with lag variables and use methods of partitioning of dynamic econometric models	Attainment of skills in using the method of least squares for construction of classical econometric models	Knowledge of the general characteristics of models with separated lags and the meaning of parameters of models with a separated lag. Using Almon's method and Koyck's method	Using the principles of defining the structure of the lag and the choice of the model type with separated lags. Using a correlogram	Construction of dynamic econometric models of different types. Choosing the type of model with separated lags. Using the Koyck's transformation	A student <b>must</b> 1) construct dynamic econometric models with autoregression, with a separated lag; 2) interpret parameters of models of autoregression and models with a separated lag

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НАВЧАЛЬНЕ ВИДАННЯ

# ЕКОНОМІКО-МАТЕМАТИЧНІ МЕТОДИ

**Робоча програма  
для студентів спеціальності  
292 "Міжнародні економічні відносини"  
першого (бакалаврського) рівня  
(англ. мовою)**

*Самостійне електронне текстове мережеве видання*

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Подано тематичний план навчальної дисципліни та її зміст за модулями й темами. Вміщено плани лекцій і практичних занять, матеріал для закріплення знань студентів (контрольні запитання, завдання для самостійної роботи), а також методику оцінювання знань студентів відповідно до вимог кредитно-трансферної системи процесу навчання.

Рекомендовано для студентів спеціальності 292 "Міжнародні економічні відносини" першого (бакалаврського) рівня.

План 2018 р. Поз. № 35 ЕВ. Обсяг 87 с.

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Видавець і виготовлювач – ХНЕУ ім. С. Кузнеця, 61166, м. Харків, просп. Науки, 9-А

*Свідоцтво про внесення суб'єкта видавничої справи до Державного реєстру  
ДК № 4853 від 20.02.2015 р.*