UDC 621.396.96
DOI: 10.15587/1729-4061.2021.229221

DEVELOPMENT OF AN OPTIMIZATION METHOD FOR MEASURING THE DOPPLER FREQUENCY OF A PACKET TAKING INTO ACCOUNT THE FLUCTUATIONS OF THE INITIAL PHASES OF ITS RADIO PULSES

Serhii Yevseiev
Doctor of Technical Sciences, Professor*
E-mail: serhii.yevseiev@hneu.net
Oleksandr Kuznietsov
PhD, Associate Professor**
Sergey Herasimov
Doctor of Technical Sciences, Professor
Department of Combat Use of Weapons of Air Defense of the Ground Forces***
Stanislav Horielyshev
PhD, Associate Professor
Research Laboratory for the Provision of Service and Military Activities of the National Guard of Ukraine
Scientific and Research Center of Service and Military Activities of the National Guard of Ukraine
National Academy of National Guard of Ukraine
Zakhysnykiv Ukrainy sq., 3, Kharkiv, Ukraine, 61001
Anton Karlov
International Military Cooperation Group***
Ihor Kovalov
PhD
Department of Special Tactics Preparation
National Academy of National Guard of Ukraine
Zakhysnykiv Ukrainy sq., 3, Kharkiv, Ukraine, 61001
Oleksii Kolomiitsev
Doctor of Technical Sciences, Senior Researcher
Department of Computer Engineering and Programming
National Technical University «Kharkiv Polytechnic Institute»
Kyrpychova str., 2, Kharkiv, Ukraine, 61002
Olena Lukashuk
PhD**
Oleksandr Milov
Doctor of Technical Sciences, Professor*
Vitaliy Panchenko
PhD, Associate Professor
National Academy of National Guard of Ukraine
Zakhysnykiv Ukrainy sq., 3, Kharkiv, Ukraine, 61001
*Department of Cyber Security and Information Technology
Simon Kuznets Kharkiv National University of Economics
Nauky ave., 9-A, Kharkiv, Ukraine, 61166
Department of Physics and Radioelectronics*
***Ivan Kozhedub Kharkiv National Air Force University
Sumska str., 77/79, Kharkiv, Ukraine, 61023

The necessity of estimating the decrease in the accuracy of measuring the informative parameters of a radar signal in real conditions of its propagation and reflection has been substantiated. The results of the estimation determine the requirements for optimizing this measurement to achieve the required efficiency. A numerical analysis of the decrease in the accuracy of measuring the Doppler frequency of a coherent packet is presented, depending on the statistical characteristics of fluctuations of the initial phases of its radio pulses. Expressions are given for calculating the fluctuation component of the measurement error of radio pulse packet frequency for various coefficients of interpulse correlation of phase fluctuations. An assessment is made of the possibility of increasing the accuracy of Doppler frequency measurement, which can be ensured by statistical optimization of the algorithm for time-frequency processing of a given radar signal by taking into account its phase fluctuations. The conditions for the multiplicative influence of phase fluctuations of radio pulses of the received packet are substantiated, which determine the efficiency of optimization of Doppler frequency measurement.

Based on the results of the study, an optimization method for measuring the Doppler frequency of the packet taking into account fluctuations in the initial phases of its radio pulses is proposed. The accuracy of Doppler frequency measurement under the influence of both the internal noise of the radar receiver and the correlated phase fluctuations of its radio pulses is estimated. The efficiency of optimization of measuring the Doppler frequency of the packet is estimated taking into account fluctuations of the initial phases of its radio pulses by means of computer simulation. It is proved that, under the influence of phase fluctuations, the accuracy of Doppler frequency measurement can be increased due to the performed optimization from 1.86 to 6.29 times. This opens the way to improving the existing algorithms for measuring the higher time range derivatives to improve the quality of tracking complex maneuvering aerodynamic objects. This explains the importance and usefulness of the work for the radar theory

Keywords: aerodynamic object, coherent packet of radio pulses, radar, RMS error, Doppler frequency.

Received date 15.02.2021
Accepted date 16.04.2021
Published date 30.04.2021

How to Cite: *Yevseiev, S., Kuznietsov, O., Herasimov, S., Horielyshev, S., Karlov, A., Kovalov, I., Kolomiitsev, O., Lukashuk, O., Milov, O., Panchenko, V. (2021). Development of an optimization method for measuring the Doppler frequency of a bucket with account for the fluctuations of the initial phases of its radio pulses. Eastern-European Journal of Enterprise Technologies, 2 (9 (110)), 6–15. doi: https://doi.org/10.15587/1729-4061.2021.229221:*

1. Introduction

Ensuring high accuracy in measuring the coordinates and motion parameters of radar surveillance objects is an urgent task for modern radars [1–3]. Thus, the maneuverability of aerodynamic objects increases, their radar visibility decreases and their ability to perform intended tasks at low and extremely low altitudes with the circumference of the terrain against the background of significant internal noise of the radar receiver increases [4–6]. This necessitates estimating their time range derivatives: radial velocity and radial acceleration with high accuracy.

In practice, a coherent packet of radio pulses is widely used to meet this requirement. In particular, the operation of measuring the radial velocity of an aerodynamic object is based on estimating the radar signal frequency. Signal frequency can be determined from the corresponding law of changes in its phase and calculated as the first time phase derivative [1].

Fluctuations of the phase front of the received signal wave, which occur due to the influence of real conditions of propagation and reflection of radio waves, lead to a violation of its coherence, which limits the accuracy of measuring informative parameters.

Currently, known statistical optimization methods for measuring radar signal parameters do not take into account the multiplicative effect of fluctuations in its phase structure. As a result, there is a need to assess the influence of the conditions of propagation and reflection of the radar signal on the decrease in the accuracy of measuring its parameters in order to further justify the feasibility of optimizing algorithms for processing the received signal. This indicates the practical usefulness of numerical evaluation of the effect of phase fluctuations of radio pulses of the received packet on the reduction of the accuracy of measuring its Doppler frequency.

Thus, the relevance of scientific research is to determine the influence of phase fluctuations of radio pulses of the received packet on the decrease in the accuracy of measuring its Doppler frequency. Appropriate statistical optimization of the measurement of phase fluctuations of radio pulses of the received packet has a practical orientation to solve the problem of tracking complex maneuvering aerodynamic objects.

2. Literature review and problem statement

In [1–4], the peculiarities of signal processing against the background of only internal receiver noises are considered. The fundamentals of radar signal processing [1] and their technical implementation, associated with the construction of appropriate devices [2], take into account only the additive interference to the useful signal and do not take into account the effect of correlated fluctuation components of the signal. Peculiarities of these issues regarding multichannel signal reception and processing are considered in [3]. The accuracy of measuring the parameters of the received signal can be considered potential. The radar signal processing algorithms presented in these works do not take into account the multiplicative interference of correlated phase fluctuations, caused by the influence of real conditions of its propagation and reflection. These fluctuations can decrease the accuracy of measuring the Doppler frequency of the radar signal due to the lack of consideration of their statistical characteristics. Attempts to perform statistical analysis on the randomness of radar signal parameters are given in [4], but the results relate only to the inspection and control of radio devices and do not apply to optimizing the measurement of unknown coordinates and motion parameters of radar objects.

The causes of such fluctuations are inhomogeneities in the Earth's atmosphere, the complex shape of the aerodynamic object and interference of direct and reflected radio waves.

Atmospheric turbulence leads to fluctuations in the signal phase due to random changes in its refractive index. Fluctuation phenomena caused by the influence of the radio wave propagation medium are considered in [1, 5–10]. Thus, in [5], a traditional mathematical apparatus is provided, which should be used to solve the research problem. In [6], the problem of substantiating the characteristics of radio waves for the detection and tracking of complex aerodynamic objects having low visibility, which is typical for modern aircrafts, is solved. In [7], the nature of signal distortions during the accelerated motion of the object in the ionosphere with respect to both single and packet radio signals is analyzed. The results of this work are certainly useful to consider as a starting point for research. However, the transformation of the signal structure considered in [7] does not take into account fluctuations in its phase due to the influence of the radio wave propagation medium. Taking into account fluctuation changes for the case of their arbitrary correlation when measuring the radial velocity and angular coordinate of the radar object is considered in [8]. However, this paper lacks a numerical analysis of possible values of statistical characteristics of phase fluctuations (variance and correlation interval) in practical cases and assessment of their impact on the accuracy of Doppler frequency measurement. To eliminate this shortcoming and take into account phase fluctuations due to the instability of atmospheric parameters, it is advisable to use data [9] and known optimization methods of space-time processing implemented in modern various-purpose radio systems [10]. For Doppler radars, such methods are provided in [10], which require further optimization taking into account the statistical characteristics of correlated phase fluctuations of the radar signal.

The complex shape of the aerodynamic object during movement leads to changes in the spatial position of its constituent elements and fluctuations in the total reflected signal. The complex shape of the air object and its ability to perform a sudden maneuver causes wandering of the radar center and, as follows, phase distortions of the received signal, which is considered in [1, 11–13]. Based on the main approaches of taking into account the motion of the surveillance object [11] and using data [12] and [13] to identify signals reflected from the elements of complex objects, it is necessary to apply the results to determine and take into account statistical characteristics of phase fluctuations caused by wandering of the radar center of the surveillance object.

Interference of direct and reflected radio waves causes phase fluctuations of the received signal due to its reflection from uneven terrain or rough sea surface. The occurrence of phase distortions during multibeam propagation of the radar signal is considered in [1–3, 5, 9, 11, 14–16]. The issues of optimizing signal reception in these conditions were considered in [14], and the features of possible frequency transformation of the signal are covered in [15]. Improvement of frequency analysis using the advantages of high-speed computational algorithms is given in [16]. Ways to improve the quality of frequency measurements were considered in [17, 18], but the issues of statistical analysis of the possibility of taking into account fluctuations in the signal phase structure under the above conditions require further research.

Thus, the above sources did not consider the assessment of the influence of phase fluctuations of the radar signal on the accuracy of measuring its informative parameters, in particular, Doppler frequency. These works lack estimation of the statistical characteristics of phase fluctuations, according to which their influence requires optimization of radar signal processing algorithms. This indicates the need for such estimation to determine the range of changes in the statistical characteristics of phase fluctuations in which their impact is the most dangerous and must be taken into account. Therefore, it is advisable to further develop these studies considering a coherent packet of radio pulses as a probing signal, the phase fluctuations of which can be described by an arbitrary correlation function.

In [19], the possibilities of optimal estimation of the packet frequency taking into account the multiplicative influence of phase fluctuations of its radio pulses are considered. However, this paper presents only the formulation of the optimal measurement problem, and only for the simplest cases of discretely decreasing and alternating laws of changes in the correlation of phase fluctuations without specifying the form of the interpulse correlation coefficient. In [20], the combined influence of internal noise of the receiver and correlated phase fluctuations of radio pulses of the received packet with an oscillating correlation function is considered. It is shown that for modern coherent-pulse radars, the fluctuation components of the RMS error of measuring the frequency of packets of (8...16) radio pulses in the troposphere can be (67.1...95.5) Hz, and in the ionosphere – (7.8...11.3), which can cause RMS errors in measuring the radial velocity of the aerodynamic object by coherent-pulse radars of several m/s. In this case, the required RMS error of measuring the radial velocity of the radar surveillance object for practical cases should be up to 1 m/s. Therefore, from a practical point of view, it is useful to further solve the problem of optimization of Doppler frequency measurement for an arbitrary form of the correlation function of phase fluctuations with the definition of statistical characteristics of phase fluctuations at which their consideration is necessary. The importance of this solution is due to the possibility of significant changes in the parameters of the correlation function of phase fluctuations due to rapid changes in radar conditions during the flight of aerodynamic objects at low and extremely low altitudes, especially above the sea surface. Therefore, it is necessary to develop an optimization method for measuring the Doppler frequency of the packet, taking into account the fluctuations of the initial phases of its radio pulses. Evaluation of the possible increase in the accuracy of measuring the Doppler shift of the packet frequency taking into account the phase fluctuations of its radio pulses will determine the efficiency of the developed optimization method.

3. The aim and objectives of the study

The study aimed to develop an optimization method for measuring the Doppler frequency of the packet to increase measurement accuracy taking into account the phase fluctuations of its radio pulses with an arbitrary correlation function.

To achieve the aim, the following objectives were set:

– to estimate values of statistical characteristics of phase fluctuations of radio pulses of the received packet, which need to be considered when measuring the Doppler frequency;

– to consider the optimization method for measuring the Doppler frequency of the packet taking into account the fluctuations of the initial phases of its radio pulses with an arbitrary correlation function;

– to evaluate the effectiveness of the optimization method for measuring the Doppler frequency of the packet, taking into account the fluctuations of the initial phases of its radio pulses.

4. Materials and methods of research

Measurement of the Doppler frequency of the coherent packet of radio pulses is considered. This type of radar signal is chosen as probing in coherent-pulse radars due to its ability to provide high resolution by Doppler frequency and high accuracy of measuring this parameter.

The centimeter-wave band of this type of radars is chosen (from 3 cm to 10 cm), in which the influence of the troposphere and complex shape of radar objects is significant. For the meter-wave band of radars (from 1 m to 2 m), the influence of ionospheric inhomogeneities and the underlying surface is significant [20].

For coherent-pulse centimeter-wave surveillance radars, the repetition rate of the packet radio pulses can vary from 680 Hz to 1,700 Hz, and meter-wave ones – from 20 Hz to 30 Hz [20]. Tracking radars use quasi-continuous sounding signals with a repetition rate of 20 kHz to 100 kHz [21].

It is believed that the phase fluctuations of the radio pulses of the received packet are distributed according to the normal law with zero mathematical expectation. In cases of radio wave propagation in real conditions, the correlation of phase fluctuations can be described by exponential or oscillating dependences [20, 22, 23].

Phase fluctuations can be caused by a combination of factors. One of the causes of phase fluctuations leading to a violation of the spatio-temporal coherence of the received signal is random inhomogeneous radio wave propagation media. Atmospheric turbulence causes fluctuations in its refractive index, which, in turn, leads to fluctuations in the parameters of radio waves propagating in the atmosphere.

In the troposphere, the fluctuations of the refractive index are due to fluctuations in temperature, pressure and humidity caused by turbulent mixing of its inhomogeneities. Inhomogeneities have different shapes and sizes, they change in time and space and move with the flow of air masses. The size of inhomogeneities varies widely. The thickness of the layers is from tenths of a meter to several hundred meters, and their horizontal dimensions – from tens of meters to tens of kilometers and more. In the troposphere, the variance of the refractive index fluctuations is $\sigma_n^2 = 0.25\ (10^{-12}...10^{-10})$ rad^2. Note that the first digit corresponds to average conditions, and the second is the maximum possible.

Taking into account the assumptions of the method of geometric optics and Gaussian correlation function of the refractive index in [7, 21], the variance of the phase fluctuation of the signal σ_φ^2, passed through the perturbed troposphere and ionosphere of the Earth. Thus, it is shown that in the troposphere, for the radar range from 3 cm to 1 m and wave path from 50 km to 200 km, the variance of the signal phase fluctuations takes values from 0.0044 rad^2 to 19.4 rad^2. In the ionosphere, the refractive index is determined by the electron concentration. Turbulent mixing of atmospheric masses, solar activity, meteors passing through the upper ionosphere layers lead to fluctuations in the electron concentration and cause random changes in the refractive index and, accordingly, the phase of the radio signal. For the radar range from 50 cm to 2 m and inhomogeneities up to 500 km, the variance of the signal phase fluctuation takes values from 0.0013 rad^2 to 8 rad^2.

Known methods of radar signal processing do not take into account the multiplicative interference of correlated phase fluctuations, caused by the influence of real conditions of its propagation and reflection. Therefore, using known optimization methods for measuring the Doppler frequency of a coherent packet of radio pulses can reduce the accuracy of measuring the Doppler frequency due to the lack of consideration of their statistical characteristics. The proposed method allows revealing the effect of phase fluctuations on the accuracy of measuring the Doppler frequency of a radio pulse packet. This is possible due to additional consideration of the values of statistical characteristics of phase fluctuations of radio pulses of the received packet when measuring the Doppler frequency.

5. Results of the development of an optimization method for measuring the Doppler frequency of the packet, taking into account fluctuations of the initial phases of its radio pulses

5. 1. Estimation of the values of statistical characteristics of phase fluctuations of radio pulses of the received packet to be considered when measuring the Doppler frequency

Random movement of a long object leads to its Doppler noise, which causes the Doppler spectrum to expand and change shape, as well as its additional shift due to changes in average radial velocity. Also, the components of the signal reflected from the rotating and oscillating elements of the object cause the occurrence of Doppler lines at frequencies that are shifted relative to the Doppler spectrum of the radar object body. In this case, the Doppler radar for the system of tracking the frequency of a certain line of the signal spectrum can capture a wrong spectral line. Phase fluctuations can also be caused by the multipath propagation of radio waves, because in the detection and ranging of low-altitude objects, the received radar signal is additionally reflected from the elements of the rough underlying surface. This leads to additional distortions of its phase structure. The variance of phase fluctuations due to these reasons can reach rad^2 units.

The results of theoretical and experimental studies indicate that phase fluctuations have a close to the normal distribution law. The correlation function of phase fluctuations can be approximated by exponential or oscillating dependences [8, 20]. The correlation interval of phase fluctuations is from tenths of a second to units of seconds, and the spectrum of phase fluctuations is low-frequency.

It is necessary to consider a possible decrease in the accuracy of Doppler frequency measurement due to phase fluctuations of the radio signal compared to the potentially possible accuracy of measuring this parameter under only internal Gaussian noise. In addition, it is necessary to determine the values of the statistical characteristics of phase fluctuations, in which their influence is predominant and should be taken into account in time-frequency processing algorithms.

The Doppler circular frequency ω of the received signal is estimated by the maximum likelihood ratio $L(\omega)$ or its natural logarithm [1, 4, 19, 20]

$$\hat{\omega} = \arg\max \ln L(\omega), \tag{1}$$

where $\omega = 2\pi(f - f_0)$ is the difference between the frequencies of the expected f and actual f_0 radar signals.

It is believed that the receiver receives a signal with random amplitude A and initial phase B, due to the additive effect of internal Gaussian noise.

The independent random nature of the amplitude and initial phase of the radar signal necessitates a separate averaging of the likelihood ratio for these parameters and transition to a double integral as follows

$$L(\omega) = \int\limits_A \int\limits_B L(\omega / A, B)\, p(A)\, p(B)\, \mathrm{d}A\mathrm{d}B, \tag{2}$$

where $L(\omega/A,B)$ is the likelihood ratio for the defined parameters A and B; $p(A)$, $p(B)$ are the corresponding probability densities of these random parameters.

The likelihood ratio determined according to (2) for the signal model with the random amplitude and the initial phase is given in [4, 19, 20].

The variance of the error of measuring the frequency of the received radio signal is determined by the maximum likelihood method [1, 4]

$$\sigma_\omega^2 = -\frac{1}{\ln L''(0)}, \tag{3}$$

where $L''(0)$ is the second derivative of the likelihood ratio (2) in the absence of a frequency mismatch between the expected and actual radar signals.

To ensure high measurement accuracy of radar signal frequency and high frequency resolution, it is advisable to use a coherent packet of radar pulses for the probing signal of the radar [1, 4, 19, 20].

The variance of the frequency measurement error (3) for a rectangular-bypass packet, according to [20], is as follows

$$\sigma_\omega^2 = \frac{12}{q^2\left[4(n/2)^2 - 1\right]T^2}, \tag{4}$$

where q^2 is the signal-to-noise ratio in terms of power; n is the number of radio pulses in the packet; T is the period of packet radio pulses.

According to research, for radio wave propagation in real conditions, the correlation of phase fluctuations can be described by exponential or oscillating dependences [19, 21] according to the expressions:

$$K_{ex}(T) = e^{-\left(\frac{T}{\tau}\right)}, \tag{5}$$

$$K_{os}(T) = e^{-\left(\frac{T}{\tau}\right)} \cos(\upsilon T), \tag{6}$$

where τ is the correlation interval of phase fluctuations; $\upsilon = 2\pi/T_{fl}$ is the oscillation frequency of the correlation coefficient of phase fluctuations; T_{fl} is the oscillation period of the correlation coefficient of phase fluctuations.

It is practical to estimate the variance of the fluctuation component of the error of measuring the Doppler frequency of the radio pulse packet σ_{fl}^2 and compare it with the noise component of this variance σ_ω^2, which is determined by expression (4). The result of this comparison allows estimating the influence of phase fluctuations on the accuracy of measuring the Doppler frequency of the radio pulse packet. This result can be expressed as the ratio of these variances $\sigma_{fl}^2/\sigma_\omega^2$ and allows determining how many times the variance of the Doppler frequency measurement error due to the correlated phase fluctuations will exceed the variance of the Doppler frequency measurement error under only internal noise.

Taking into account the results [19, 20], for the exponential correlation coefficient of phase fluctuations (5), the variance of the fluctuation component of the error of measuring the Doppler frequency of the rectangular-bypass packet is determined by the expression

$$\sigma_{fl}^2=\frac{18{\sigma_\varphi}^2}{(n/2)^2[4(n/2)^2-1]^2T^2}\times\left[\begin{array}{l}\sum\limits_{k=1}^{n/2}(2k-1)^2\left\{1-K_{ex}\left[T(2k-1)\right]\right\}+\\ +2\sum\limits_{l=1}^{n/2-k}K_{ex}(Tl)\sum\limits_{k=1}^{n/2-1}\begin{array}{l}(2k-1)(2k+2l-1)\times\\ \times\left\{1-K_{ex}\left[T(2k-1)\right]\right\}\end{array}\end{array}\right], \tag{7}$$

where σ_φ^2 is the variance of phase fluctuations of the packet radio pulses.

According to [20], for phase fluctuations described by the oscillating correlation coefficient (6), the specified variance is determined as follows

$$\sigma_{fl}^2=\frac{18{\sigma_\varphi}^2}{(n/2)^2\left[4(n/2)^2-1\right]^2T^2}\times\left[\sum_{k=1}^{n/2}(2k-1)^2\left[1-K_{os}(2k-1)\right]\right]+2\sum_{l=1}^{n/2-k}K_{ex}(Tl)\times\left\{\begin{array}{l}\sum\limits_{k=1}^{n/2-1}(2k-1)(2k+2l-1)\times\\ \times\left(\cos(k\upsilon T)-K_{ex}(2k-1)\cos\left[(2k+l-1)\upsilon T\right]\right)\end{array}\right\}. \tag{8}$$

The obtained expressions (7), (8) allow determining the variance of the error of measuring the Doppler frequency of the packet without taking into account the correlated phase fluctuations of its radio pulses.

Taking into account expressions (4), (7), (8), the ratio $\sigma_{fl}^2/\sigma_\omega^2$ will be determined for the exponential correlation coefficient of phase fluctuations (5) by expression

$$\sigma_{fl}^2/\sigma_\omega^2=\frac{3q^2{\sigma_\varphi}^2}{2(n/2)^2\left[4(n/2)^2-1\right]}\times\left[\begin{array}{l}\sum\limits_{k=1}^{n/2}(2k-1)^2\left\{1-K_{ex}\left[T(2k-1)\right]\right\}+\\ +2\sum\limits_{l=1}^{n/2-k}K_{ex}(Tl)\sum\limits_{k=1}^{n/2-1}\begin{array}{l}(2k-1)(2k+2l-1)\times\\ \times\left\{1-K_{ex}\left[T(2k-1)\right]\right\}\end{array}\end{array}\right], \tag{9}$$

and for the oscillating correlation coefficient of phase fluctuations (5) by expression

$$\sigma_{fl}^2/\sigma_\omega^2=\frac{3q^2{\sigma_\varphi}^2}{2(n/2)^2\left[4(n/2)^2-1\right]}\times\left\{\sum_{k=1}^{n/2}(2k-1)^2\left[1-K_{os}(2k-1)\right]\right\}+2\sum_{l=1}^{n/2-k}K_{ex}(Tl)\times\left\{\begin{array}{l}\sum\limits_{k=1}^{n/2-1}(2k-1)(2k+2l-1)\times\\ \times\left(\cos(k\upsilon T)-K_{ex}(2k-1)\cos\left[(2k+l-1)\upsilon T\right]\right)\end{array}\right\}. \tag{10}$$

Coherent processing of radio pulse sequences in most modern radars provides coherent accumulation of packets of up to 20 radio pulses. At the same time, their number is even, which is due to the peculiarities of digital processing of radar signals and the implementation of discrete and fast Fourier transform algorithms. Therefore, the analysis is performed for coherent packets of (8...16) radar pulses, which are widely used in radar practice. According to [1, 17, 21], the influence of the above factors of phase fluctuations distorts the correlation interval of these fluctuations $=(0.1...1)$ s. The variance of phase fluctuations for estimating their impact is chosen within $\sigma_\varphi^2=(0.01...10)$ rad^2 depending on the radar range. The estimation should be performed for the resolution coefficient $K_r=10\lg(q^2/2)$ of 17-27 db. These data are typical for surveillance and tracking radars and are confirmed by detection quality estimates given, for example, in [21]. Taking into account the characteristics of coherent-pulse radars, the ratio of the period of radio pulses of the received packet to the correlation interval of phase fluctuations can be in the range $T/\tau=(10^{-5}...10^{-2})$. According to expressions (5), (6), this corresponds to a decrease in the correlation coefficient of phase fluctuations by 1 %, which occurs under normal radar operating conditions.

Fig. 1 shows graphs of the ratio $\sigma_{fl}^2/\sigma_\omega^2$ against T/τ according to expression (10).

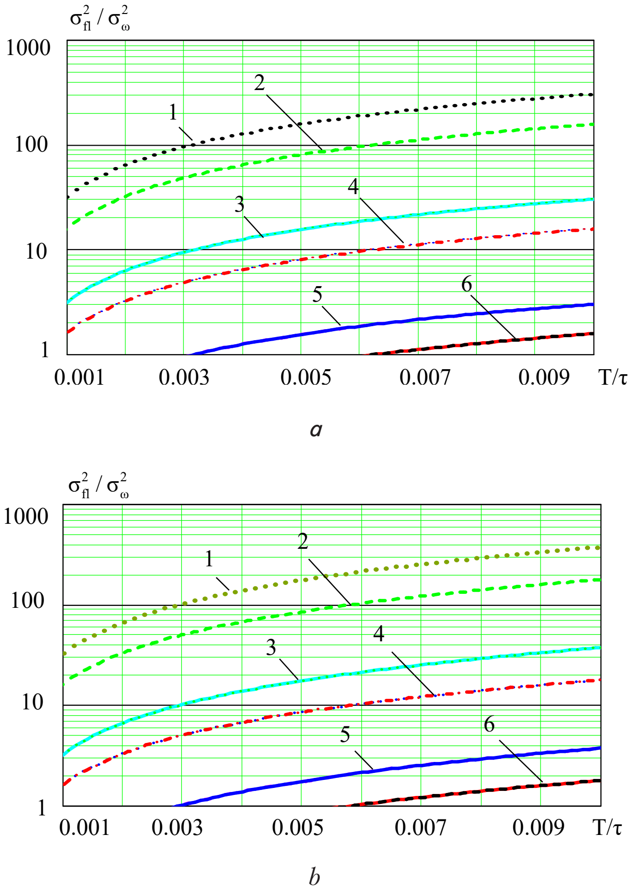
The graphs in Fig. 1, *a* were obtained according to expression (9) for the phase correlation coefficient (5) and correspond to the following cases: 1 – n=16; K_r=27 db; $\sigma_\varphi^2=10$ rad^2; 2 – n=8; K_r=27 db; $\sigma_\varphi^2=10$ rad^2; 3 – n=16; K_r=17 db; $\sigma_\varphi^2=10$ rad^2 and n=16; K_r=27 db; $\sigma_\varphi^2=1$ rad^2; 4 – n=8; K_r=17 db; $\sigma_\varphi^2=10$ rad^2 and n=8; K_r=27 db; $\sigma_\varphi^2=1$ rad^2; 5 – n=16; K_r=17 db; $\sigma_\varphi^2=1$ rad^2 and n=16; K_r=27 db; $\sigma_\varphi^2=0.1$ rad^2; 6 – n=8; K_r=17 db; $\sigma_\varphi^2=1$ rad^2; and n=8; K_r=27 db; $\sigma_\varphi^2=0.1$ rad^2.

In Fig. 1, *b*, the graphs correspond to similar conditions in Fig. 1, *a* and were obtained for the phase correlation coefficient (10) with the oscillation period selected for the condition $T_{fl}/\tau=3$, as confirmed by experimental studies [21].

First, it is advisable to analyze the influence of the energy characteristics of the coherent radio pulse packet on the ratio $\sigma_{fl}^2/\sigma_\omega^2$. An increase in the resolution coefficient from 17 db to 27 db causes a corresponding 10-fold increase in the ratio $\sigma_{fl}^2/\sigma_\omega^2$, which can be seen when comparing curves 1 and 3; 2 and 4; 3 and 5; 4 and 6 (for phase correlation coefficients (5) – Fig. 1, *a* and (6) – Fig. 1, *b*).

The transition from coherent accumulation of the packet with n=8 radio pulses to coherent accumulation of the packet with n=16 radio pulses causes a 1.9-fold increase in the ratio $\sigma_{fl}^2/\sigma_\omega^2$ according to the comparison of curves 1 and 2, 3 and 4, 5 and 6 for the phase correlation coefficient (5) – Fig. 1, *a* and 2.1-fold for similar curves at the phase correlation coefficient (6) – Fig. 1, *b*.

The analysis of the influence of statistical characteristics of phase fluctuations of radio pulses of the received packet on the variance ratio $\sigma_{fl}^2/\sigma_\omega^2$ is considered below. The increase in the variance of phase fluctuations σ_φ^2 from 0.1 rad^2 to 1 rad^2 and from 1 rad^2 to 10 rad^2, i.e. by an order of magnitude, causes an increase in the ratio $\sigma_{fl}^2/\sigma_\omega^2$ also by an order of magnitude, as can be seen by comparing curves 4 and 6; 3 and 5; 1 and 3; 2 and 4 (for phase correlation coefficients according to expression (5) – Fig. 1, *a* and according to expression (6) – Fig. 1, *b*).



a

b

Fig. 1. Graphs of $\sigma_{fl}^2/\sigma_\omega^2$ against T/τ for determining the ratio $\sigma_{fl}^2/\sigma_\omega^2$: *a* – according to expression (9); *b* – according to expression (10)

5. 2. Consideration of the optimization method for measuring the Doppler frequency of the packet taking into account fluctuations of the initial phases of its radio pulses with an arbitrary correlation function

The predominance of the influence of correlated phase fluctuations of the packet radio pulses over the influence of internal noise of the receiver is considered. With a sufficiently strong radio signal, the accuracy of Doppler frequency measurement is determined by the upper part of the maximum of the time-frequency mismatch function. In the presence of radio signal phase fluctuations, the maximum of this function expands and deviates from the origin, which reduces the accuracy of Doppler frequency measurement. Optimization consists in finding the likelihood ratio taking into account the statistical characteristics of phase fluctuations of radio pulses of the received packet radio signal, described by an arbitrary correlation function. This consideration minimizes the variance of the fluctuation component of the Doppler frequency measurement error and, as a result, increases measurement accuracy.

We believe that the coherent packet of radio pulses partially lost coherence due to atmospheric inhomogeneities, Earth's (sea) surface and reflective properties of radar objects.

The presence of fluctuation components φ_k (k=1,2…n is the radio pulse number) in the phases of the received packet radio pulses requires additional averaging of the likelihood ratio $L(\omega)$ (2) according to these fluctuation components. Using the method described in [19], for packets with the symmetric distribution of radio pulse amplitudes, the result of this averaging is as follows

$$\bar{L}_{opt}(\omega)=\int_{\vec{\varphi}} L(\omega)\,p(\Delta\vec{\varphi})\,\mathrm{d}\Delta\vec{\varphi}, \tag{11}$$

where $\vec{\varphi}=\left\|\varphi_k\right\|$ is the vector of values of the fluctuation components of the phases of the packet radio pulses; $p(\Delta\vec{\varphi})$ is the distribution of differences of fluctuation components of radio pulse phases symmetric relative to the packet center $d\Delta\vec{\varphi}=d\Delta\varphi_1 d\Delta\varphi_2 \ldots d\Delta\varphi_{n/2}$.

The problem of optimizing the measurement of the angular Doppler frequency of the radio pulse packet can be solved by determining the maximum argument of the natural logarithm of the likelihood ratio (11) $\bar{L}_{opt}(\omega)$:

$$\hat{\omega}_{opt}=\arg\max\ln\bar{L}_{opt}(\omega). \tag{12}$$

Therefore, information about the optimal packet frequency is contained in the phase difference of its symmetrical radio pulses.

In this case, as shown in [7], the arbitrariness of the correlation coefficient of phase fluctuations can be taken into account by introducing the matrix

$$\bar{M}=$$

$$=\begin{pmatrix} 1 & K_1(T) & K_2(T) & \ldots & K_{n-2}(T) & K_{n-1}(T) \\ K_1(T) & 1 & K_1(T) & \ldots & K_{n-3}(T) & K_{n-2}(T) \\ K_2(T) & K_1(T) & 1 & \ldots & K_{n-4}(T) & K_{n-3}(T) \\ . & . & . & . & . & . \\ K_{n-2}(T) & K_{n-3}(T) & K_{n-4}(T) & \ldots & 1 & K_1(T) \\ K_{n-1}(T) & K_{n-2}(T) & K_{n-3}(T) & \ldots & K_1(T) & 1 \end{pmatrix}, \tag{13}$$

where K_{n-i} is the arbitrary correlation coefficient of phase fluctuations of the packet radio pulses, spaced in the time interval $(n-i)T$.

According to [7], the law of distribution of differences in the fluctuation components of phases of symmetric packet radio pulses for an arbitrary correlation coefficient of phase fluctuations is as follows

$$p\left(\Delta\varphi_1,\Delta\varphi_2,\ldots,\Delta\varphi_{n/2}\right)=\frac{D^{\frac{n/2-1}{2}}}{\left(2\sigma_\varphi\right)^{n/2}\pi^{n/4}\sqrt{|Q|}}\times$$

$$\times\exp\left[-\frac{1}{4\sigma_\varphi^2 D}\sum_{l=1}^{n/2}\sum_{k=1}^{n/2}\left(D_{n/2+1-l,n/2+1-k}-D_{n/2+1-k,n/2+l}\right)\Delta\varphi_l\Delta\varphi_k\right], \tag{14}$$

$$\Delta\varphi_l = \varphi_{n/2+1-l} - \varphi_{n/2+l}, \tag{15}$$

$$\Delta\varphi_k = \varphi_{n/2+1-k} - \varphi_{n/2+k}, \tag{16}$$

where $D = \left|\bar{M}\right|$; $D_{i,k}$ is the algebraic complement (minor) of the element $M_{i,k}$ of the matrix $\bar{M}$; $\Delta\varphi_l$, $\Delta\varphi_k$ are the differences of the fluctuation components of the phases of radio pulses of the l-th and k-th symmetric pair, respectively; $|Q|$ is the determinant of the matrix of coefficients with second-degree quadratic members under the double summation sign.

The determinant $|Q|$ is as follows

$$|Q| = \begin{vmatrix} D_{n/2,n/2} + D_{n/2,n/2+1} & . & . & D_{1,n/2} + D_{1,n/2+1} \\ D_{n/2-1,n/2} + D_{n/2-1,n/2+1} & . & . & D_{1,n/2-1} + D_{1,n/2+2} \\ . & . & . & . \\ D_{1,n/2} + D_{1,n/2+1} & . & . & D_{1,1} + D_{1,n} \end{vmatrix}. \tag{17}$$

The obtained relations (11)–(17) represent the optimization method for measuring the Doppler frequency of the packet taking into account fluctuations of the initial phases of its radio pulses with an arbitrary correlation function.

We evaluate the effectiveness of the optimization method for measuring the Doppler frequency of the packet taking into account the correlated phase fluctuations of its radio pulses by evaluating the possibilities of improving the accuracy of packet Doppler frequency measurement.

5. 3. Evaluation of the efficiency of the optimization method for measuring the Doppler frequency of the packet taking into account fluctuations of the initial phases of its radio pulses

Finding the likelihood ratio (11) averaged by the fluctuation components of the packet radio pulse phases allows us to proceed to estimating the error variance of the optimal measurement of the Doppler frequency of the received packet taking into account the phase fluctuations of its radio pulses. Comparison of this variance with the variances of the Doppler frequency measurement error without taking into account the phase fluctuations of the packet radio pulses, determined by expressions (7), (8), allows proving the effectiveness of the proposed optimization method for Doppler frequency measurement.

Similar to expression (3), the variance of the error of the optimal measurement of the frequency of the received packet, taking into account the phase fluctuations of its radio pulses can be obtained according to the expression

$$\sigma^2_{\omega opt} = -\frac{1}{\ln \bar{L}''_{opt}(0)}, \tag{18}$$

where $\bar{L}''_{opt}(0)$ is the second derivative of the likelihood ratio (11) in the absence of frequency mismatch.

If the phase fluctuations are significant and their influence significantly exceeds that of the internal noise of the receiver, expression (18) has the following solution:

$$\sigma^2_{\omega opt} = \frac{2\sigma^2_\varphi\left[1+K(T)\right]}{T^2\left[1-K(T)\right](n-1)} \times$$
$$\times \left[\frac{\left[1-K(T)\right]^2}{\left[4(n-1)^2-1\right]\left[1-K(T)\right]^2+\left[n-1-(n-3)K(T)\right]}\right]. \tag{19}$$

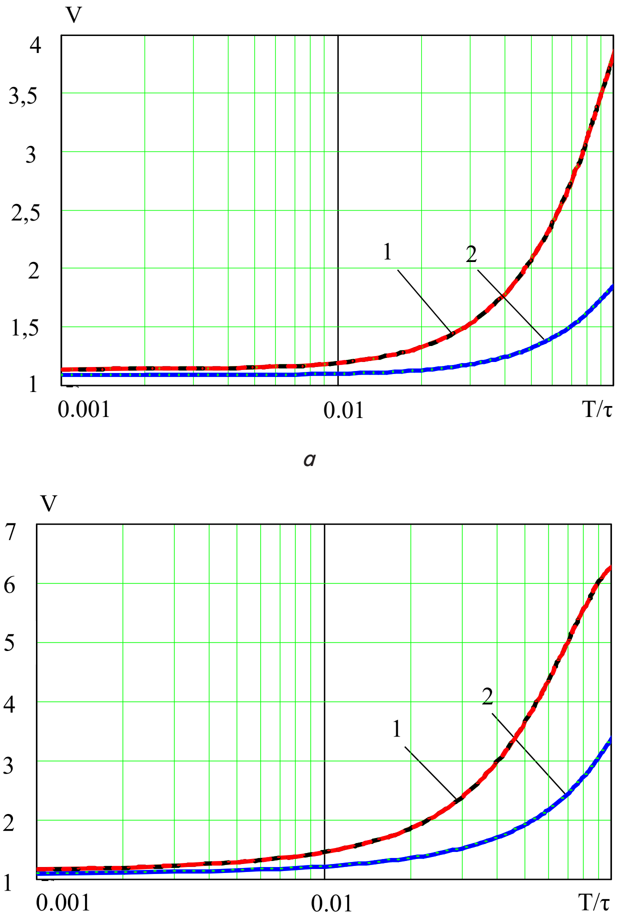
Expression (19) is obtained for an arbitrary correlation coefficient of phase fluctuations.

The efficiency of optimization of packet Doppler frequency measurement taking into account the phase fluctuations of radio pulses is proposed to be evaluated using the ratio

$$V = \frac{\sigma^2_{fl}}{\sigma^2_{\omega opt}}. \tag{20}$$

To evaluate the possible efficiency of the proposed optimal algorithm, it is advisable to apply expression (20) for the correlation coefficients of phase fluctuations according to expressions (5), (6), which, respectively for σ^2_{fl}, are determined by formulas (7), (8).

Fig. 2 provides a graph of efficiency V against the ratio T/τ, if the phase fluctuations of the radio pulses of the received packet are characterized by the above correlation coefficients (5) – Fig. 2, *a* and (6) – Fig. 2, *b*.



a

b

Fig. 2. Efficiency of optimization of radio pulse packet frequency measurement for phase fluctuations with correlation coefficients: *a* — according to expression (5); *b* —according to expression (6)

In the graphs of Fig. 2, *a*, curve 1 is obtained for n=16 under the following conditions:

1) K_r=27 db, $\sigma^2_\varphi = 10$, 1, 0.1 rad^2;

2) K_r=17 db, $\sigma^2_\varphi = 10$, 1 rad^2, which corresponds to curves 1, 3 and 5 in Fig. 1, *a*. Curve 2 is obtained for n=8 un-

der similar conditions and corresponds to curves 2, 4 and 6 in Fig. 1, *a*. The curves shown in Fig. 2, *b* have the same correspondence to the curves shown in Fig. 1, *b*.

Normal radar operating conditions correspond to $T/\tau=(10^{-5}...10^{-2})$. For such conditions, the efficiency of taking into account phase fluctuations with exponential and oscillating correlation functions is insignificant: 1.1...1.19 times (Fig. 2, *a*), and 1.21...1.45 times (Fig. 2, *b*).

Conditions of significant influence of phase fluctuations due to the perturbed Earth's troposphere or ionosphere, significant influence of the Earth's (sea) surface and complex maneuvering of radar objects correspond to $T/\tau=(10^{-2}...10^{-1})$. In this case, the efficiency can reach 1.86...3.86 times (Fig. 2, *a*), and 3.39...6.29 times (Fig. 2, *b*).

These results indicate that the most significant factor determining the feasibility of optimizing the time-frequency processing of the radar signal is to assess the degree of temporal decorrelation of its phase.

Fig. 3 shows the change in the exponential (curve 1) and oscillating (curve 2) correlation coefficient of phase fluctuations within the ratio $T/\tau=(10^{-2}...10^{-1})$.

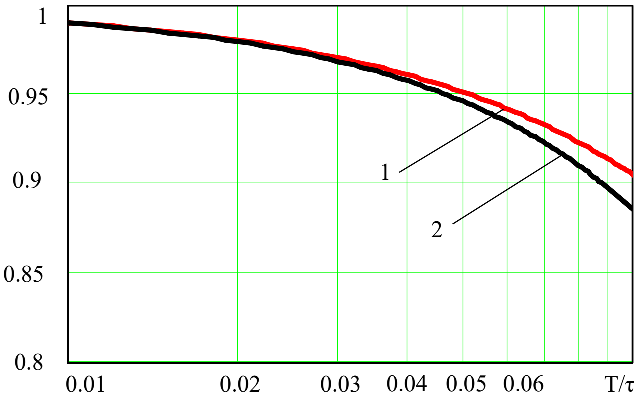


Fig. 3. Change of the correlation coefficient of phase fluctuations

6. Discussion of the results of evaluating the effectiveness of the optimization method for measuring the Doppler frequency of the received packet

Let's analyze the obtained graphs (Fig. 1–3). For the increase in the ratio of the received packet radio pulse period to the correlation interval of phase fluctuations T/τ to 10^{-2} (decrease of the phase correlation coefficient by 1 %). The ratio $\sigma_{fl}^2/\sigma_\omega^2$ (Fig. 1, *a*) increases several times: 300 (curve 1); 157 (curve 2); 30 (curve 3); 15 (curve 4); 3 (curve 5) and 1.5 (curve 6). For the phase correlation coefficient (11), this increase is several times: (Fig. 1, *b*): 373 (curve 1); 177 (curve 2); 37 (curve 3); 17 (curve 4); 4 (curve 5) and 1.7 (curve 6). Fluctuations of the phase front of the radar signal wave cause the variance of the fluctuation component of the packet frequency measurement error σ_{fl}^2 to exceed the variance σ_ω^2 of the noise component of this error hundreds of times. This is true for fluctuations σ_ω^2 under real conditions of propagation and reflection of the radar signal, i.e. for the ratio T/τ in the range $(10^{-3}...10^{-2})$. In this case, the greatest influence on the value of the studied ratio $\sigma_{fl}^2/\sigma_\omega^2$ is exerted by the statistical characteristics of phase fluctuations – variance σ_φ^2 and phase correlation coefficient K(T). For the exponential (5) and oscillating (6) correlation coefficients of phase fluctuations $\sigma_{fl}^2/\sigma_\omega^2$, the ratio takes close values.

With the variance of phase fluctuations $\sigma_\varphi^2=(1...10)\ \text{rad}^2$ and increase in the ratio T/τ to 10^{-2} and more, the influence of phase fluctuations of the radio pulses of the received packet can be considered dominating over the influence of internal noise. At the same time, RMS errors of measuring the radial velocity of radar surveillance objects, caused by the influence of the troposphere for centimeter-wave radars and ionosphere for meter-wave radars, can reach units of m/s.

Radar surveillance of air objects is often accompanied by changes in detection and ranging conditions, which in turn leads to changes in the correlation law of phase fluctuations. This requires optimization of measurement of the radio pulse packet Doppler frequency taking into account phase fluctuations with their arbitrary correlation function.

As shown by the evaluation results in Fig. 1, 2, even with high phase fluctuations $\sigma_\varphi^2=(1...10)\ \text{rad}^2$, their high time correlation $T/\tau=(10^{-5}...10^{-2})$ ($K(T)>0.99$) causes a possible increase in the accuracy of Doppler frequency measurement only by one to tens of percent.

The graphs in Fig. 3 show that for $T/\tau\leq10^{-2}$, the values of the phase correlation coefficients (5) – curve 1 and (6) – curve 2 almost coincide $K(0.01)$~0.99.

For significant time decorrelation of the phase of the received packet radio pulses, the ratio interval is $T/\tau=(10^{-2}...10^{-1})$. The values of the phase correlation coefficients (5), (6) decrease significantly and begin to differ, $K(0.1)$~0.9 and $K(0.1)$~0.88, respectively.

In the defined range $T/\tau=(10^{-2}...10^{-1})$, there is a significant increase in efficiency according to expression (20) several times. This is due to taking into account the nature of changes in the correlation of phase fluctuations, which proves the feasibility of optimizing the time-frequency processing of the radar signal in these conditions.

The limitation of this study is that the radar estimates the average radial velocity when detecting and ranging low-altitude objects. As noted above, the radar signal is additionally reflected from the elements of the rough underlying surface and causes phase fluctuations.

The obtained results can be further developed in the direction of optimizing algorithms of spatial processing of the radar signal in order to increase the accuracy of measuring the angular coordinates of aerodynamic objects. This task is practically important for radars that provide altitude measurements under tropospheric refraction conditions [22]. At the same time, taking into account correlated phase fluctuations becomes especially important for phased-array radars, which implement the phase method of determining angular coordinates.

7. Conclusions

1. The values of statistical characteristics of phase fluctuations of radio pulses of the received packet are estimated, according to which their accounting when measuring the Doppler frequency is necessary. The results were obtained for packets with a typical number of radio pulses (8...16) used in coherent-pulse radars. The ranges of variance and correlation interval of phase fluctuations are determined, according to which the excess of the variance of the fluctuation component of the packet frequency measurement

error over the variance of the noise component of this error reaches hundreds of times, which determines the conditions for taking into account phase fluctuations in algorithms for coherent processing of a given radar signal. It was found that this occurs when the variance of phase fluctuations changes in the range from 1 rad^2 to 10 rad^2 and the ratio of the packet radio pulse period to the correlation interval of phase fluctuations of the signal of 10^{-2} and higher.

2. The optimization method for measuring the Doppler frequency of the packet taking into account fluctuations of initial phases of radio pulses with an arbitrary correlation function is considered. Optimization consists in finding the likelihood ratio taking into account the law of distribution of differences of fluctuation components of phases of symmetric radio pulses of the packet for an arbitrary correlation coefficient of phase fluctuations. This optimization minimizes the variance of the fluctuation component of the Doppler frequency measurement error.

3. The efficiency of the optimization method for measuring the Doppler frequency of the packet taking into account fluctuations of initial phases of radio pulses is estimated. It is proved that under the influence of phase fluctuations, the accuracy of Doppler frequency measurement due to the performed optimization can be increased from 1.86 to 6.29 times. It is determined that the most significant factor determining the feasibility of optimizing the time-frequency processing of the radar signal is the degree of temporal decorrelation of its phase.

Acknowledgments

The obtained results are based on the scientific achievements of the Doctor of Technical Sciences Professor Mykola Mykolayovych Minervin and an outstanding scientist in the field of radar theory, Doctor of Technical Sciences Professor Yakov Davydovych Shirman.

The author wishes to express special gratitude to the Head of the Department of Physics and Radio Electronics of the Ivan Kozhedub Kharkiv National Air Force University, Honored Worker of Science and Technology of Ukraine, Doctor of Technical Sciences, Professor Volodymyr Dmytrovych Karlov for consulting and practical assistance during the study.

Reference

1. Zohuri, B. (2020). Fundaments of Radar. Radar Energy Warfare and the Challenges of Stealth Technology, 1–110. doi: https://doi.org/10.1007/978-3-030-40619-6_1
2. Melvin, W. L., Scheer, J. (Eds.) (2012). Principles of Modern Radar: Advanced techniques. IET. doi: https://doi.org/10.1049/sbra020e
3. Klemm, R., Nickel, U., Gierull, C., Lombardo, P., Griffiths, H., Koch, W. (Eds.) (2017). Novel Radar Techniques and Applications Volume 1: Real Aperture Array Radar, Imaging Radar, and Passive and Multistatic Radar. IET. doi: https://doi.org/10.1049/sbra512f
4. Herasimov, S., Roshchupkin, E., Kutsenko, V., Riazantsev, S., Nastishin, Yu. (2020). Statistical analysis of harmonic signals for testing of Electronic Devices. International Journal of Emerging Trends in Engineering Research, 8 (7), 3791–3798. doi: https://doi.org/10.30534/ijeter/2020/143872020
5. Barton, D. K. (2012). Radar Equations for Modern Radar. Artech House, 264.
6. Herasimov, S., Belevshchuk, Y., Ryapolov, I., Volkov, A., Borysenko, M., Tokar, O. (2020). Modeling technology of radar scattering of the fourth generation EF-2000 Typhoon multipurpose aircraft model. International Journal of Emerging Trends in Engineering Research, 8 (9), 5075–5082. doi: https://doi.org/10.30534/ijeter/2020/30892020
7. Minervin, N. N., Karlov, D. V., Konovalov, V. M. (2013). Features of influencing the ionosphere on radar signals at accelerated motion of space objects. Applied Radio Electronics, 12 (4), 530–532.
8. Minervin, N. N., Kuznetsov, A. L. (2013). Optimal algorithms for measuring target radial velocity and received signal arrival angle in view of phase fluctuations with arbitrary correlation function. Applied Radio Electronics, 12 (4), 514–517.
9. Volosyuk, V. K., Gulyaev, Y. V., Kravchenko, V. F., Kutuza, B. G., Pavlikov, V. V., Pustovoit, V. I. (2014). Modern methods for optimal spatio-temporal signal processing in active, passive, and combined active-passive radio-engineering systems. Journal of Communications Technology and Electronics, 59 (2), 97–118. doi: https://doi.org/10.1134/s1064226914020090
10. Klochko, V. K. (2016). Algorithms of 3D radio-wave imaging in airborne Doppler radar. Radioelectronics and Communications Systems, 59 (8), 335–343. doi: https://doi.org/10.3103/s0735272716080021
11. Richards, M. A. (2014). Fundamentals of Radar Signal Processing. McGraw-Hill Education.
12. O'Neill, C. R., Arena, A. S. (2005). Time Domain Training Signals Comparison for Computational Fluid Dynamics Based Aerodynamic Identification. Journal of Aircraft, 42 (2), 421–428. doi: https://doi.org/10.2514/1.6424
13. Singh, M., Bhoi, S. K., Khilar, P. M. (2017). Short-Range Frequency-Modulated Continuous Wave (FMCW) Radar Using Universal Software-Defined Radio Peripheral (USRP). Progress in Intelligent Computing Techniques: Theory, Practice, and Applications, 559–565. doi: https://doi.org/10.1007/978-981-10-3376-6_60
14. Wu, X., Tian, Z., Davidson, T., Giannakis, G. (2006). Optimal waveform design for UWB radios. IEEE Transactions on Signal Processing, 54 (6), 2009–2021. doi: https://doi.org/10.1109/tsp.2006.872556

15. Karimi-Ghartemani, M., Iravani, M. R. (2005). Measurement of harmonics/inter-harmonics of time-varying frequencies. IEEE Transactions on Power Delivery, 20 (1), 23–31. doi: https://doi.org/10.1109/tpwrd.2004.837674
16. Valenzuela, J., Pontt, J. (2009). Real-time interharmonics detection and measurement based on FFT algorithm. 2009 Applied Electronics, 259–264.
17. Tian, X., Zhang, T., Zhang, Q., Xu, H., Song, Z. (2018). Pulse Compression Analysis for OFDM-Based Radar-Radio Systems. Machine Learning and Intelligent Communications, 381–390. doi: https://doi.org/10.1007/978-3-319-73447-7_42
18. Herasimov, S., Tymochko, O., Kolomiitsev, O., Aloshin, G., Kriukov, O., Morozov, O., Aleksiyev, V. (2019). Formation Analysis of Multi-Frequency Signals of Laser Information Measuring System. EUREKA: Physics and Engineering, 5, 19–28. doi: https://doi.org/10.21303/2461-4262.2019.00984
19. Karlov, V., Kuznietsov, O., Artemenko, A. (2018). Statement of problem of target's radial velocity optimal estimation using initial phases correlating fluctuations of received radio pulses bursts. Zbirnyk naukovykh prats Kharkivskoho natsionalnoho universytetu Povitrianykh Syl, 3, 115–121. doi: https://doi.org/10.30748/zhups.2018.57.17
20. Kuznietsov, O., Karlov, V., Karlov, A., Kiyko, A., Lukashuk, O., Biesova, O., Petrushenko, M. (2020). Estimation of the Dispersion of the Error in Measuring the Frequency of a Pack with Correlated Fluctuations in the Initial Phases of its Radio Pulses. 2020 IEEE Ukrainian Microwave Week (UkrMW). doi: https://doi.org/10.1109/ukrmw49653.2020.9252588
21. Siedyshev, Yu. M., Karpenko, V. I., Atamanskyi, D. V. et. al. (2010). Radioelektronni systemy. Kharkiv: KhUPS, 418.
22. Mogyla, A. A. (2014). Application of stochastic probing radio signals for the range-velocity ambiguity resolution in doppler weather radars. Radioelectronics and Communications Systems, 57 (12), 542–552. doi: https://doi.org/10.3103/s0735272714120036
23. Ghasemi, A., Abedi, A., Ghasemi, F. (2012). Propagation of Radar Waves. Propagation Engineering in Radio Links Design, 299–365. doi: https://doi.org/10.1007/978-1-4614-5314-7_6