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AI-BASED OPTIMIZATION OF PACKING PROBLEMS FOR ENHANCING MEDICAL SAFETY SYSTEMS

Abstract. In the context of enhancing medical safety systems, AI-based optimization of packing solutions is crucial, particularly in the secure and sanitary storage of hazardous materials within healthcare environments. Ensuring that toxic, radioactive, or infectious substances are stored according to strict sanitary guidelines is vital for protecting public health and preventing contamination. This study introduces an AI-driven mathematical model to address packing problems, with a focus on meeting both technological and hygiene standards. By modeling the optimal placement of containers in a deployment area and incorporating key safety and sanitary restrictions, the problem is reduced to a nonlinear programming framework. The phi-function technique is utilized to capture geometric relationships effectively, enabling the development of AI-optimized, sanitary-compliant storage solutions. Numerical examples are provided to demonstrate the approach's efficacy.

Keywords: AI-based optimization, medical safety, packing problem, safety clearance, deployment area, mathematical modelling, non-linear programming.

Introduction

In an era of rapid technological advancement, the development and deployment of safety systems are critical to ensuring the secure and efficient operation of various systems, particularly in healthcare environments [1]. These systems are essential for safeguarding both human life and infrastructure, playing a pivotal role in mitigating risks and preventing potentially catastrophic outcomes. Nowhere is this more evident than in medical institutions, where the failure of safety systems can have severe consequences, including the spread of infectious agents or exposure to toxic and radioactive materials [2].

In healthcare settings, safety systems must be meticulously designed not only to prevent accidents but also to mitigate the impact of any incidents. Modern systems, leveraging continuous monitoring and advanced *AI*-driven analytics, are capable of identifying potential risks in real time and triggering prompt response mechanisms. However, a critical aspect of medical safety often overlooked is the proper storage of hazardous substances, such as toxic, radioactive, or infectious materials, where both technological and sanitary compliance are necessary to prevent contamination and ensure swift response in emergencies [3].

Previous studies have highlighted the risks associated with incorrect placement and storage of hazardous materials in healthcare environments [4]. Safety considerations, such as determining safe distances, are often addressed too late in the project lifecycle, leading to inefficiencies and increased risk.

This paper seeks to address these challenges through an *AI*-based approach, offering optimization techniques that enhance medical safety by ensuring the strategic and sanitary-compliant placement of hazardous materials in healthcare facilities.

Recent Research and Publications

The problem of finding the optimal placement of containers, considering given restrictions, can be formulated as a packing problem [5,6]. The theory of optimization geometric design is one of the tools for studying and optimizing complex technical systems to achieve their optimal functioning state. It is designed to solve a number of applied optimization problems of placing geometric objects. These problems are associated with the creation of energy- and resource-saving technologies in priority sectors of the national economy (energy, machine-, ship-, aircraft-building, construction, chemical industry, as well as in scientific research in the field of nanotechnologies, in modern tasks of biology, mineralogy, medicine, materials science, in robotics, tasks of coding information, systems of image recognition, control systems of spacecraft) during the automation and modeling of the processes of placing various objects [7,8].

Several heuristic approaches have been developed to address the problem of arranging equal-sized circles within a circular container [9,10]. This problem, like many packing problems, involves finding the most efficient arrangement of a fixed set of objects to minimize the overall dimensions of the container. A range of studies has approached this issue through nonlinear models and optimization techniques. In particular, numerous nonlinear programming models have been explored for minimizing the dimensions of containers in 2*D* packing problems, which often involve various shapes, such as squares, rectangles, triangles, and circles.

The goal of this research is to develop a precise mathematical model for optimizing placement of containers in a deployment area and to devise efficient methods for solving this problem. This paper introduces an intelligent system designed to determine the optimal arrangement of containers within a storage area. By framing the problem as one of optimizing the placement of congruent circles in a multiconnected domain, while adhering to safety clearance, we address a critical challenge in the fields of medical safety.

The system employs advanced mathematical techniques, particularly the phi-function method, to accurately describe the geometric relationships between containers. This method simplifies the problem by transforming it into a nonlinear programming model.

By integrating these mathematical models and algorithms, the intelligent system provides a robust solution for ensuring both safe and efficient storage. Its key features and benefits are highlighted, underscoring its potential to improve safety protocols and enhance storage efficiency across a wide range of facilities. Moreover, the system is particularly relevant to healthcare environments, where its *AI*-based optimization capabilities can significantly enhance medical safety systems by ensuring that hazardous materials are stored securely and in compliance with sanitary guidelines.

Problem formulation

The primary aim of optimization packing problem can be described as follows: to search for a spatial arrangement of a given set of geometric objects within a deployment area that satisfies safety clearance, while maximizing or minimizing a specific optimization goal.

To address the problem in relation to safety systems within the framework of placement problem, it is essential to define key elements of the system, such as:

- the shape and boundaries of the deployment area;
- the geometry of the objects to be placed in this area;
- the technological and safety constraints governing the arrangement of these objects within the designated space;
- the optimization objective.

The area may have a complex shape, with certain regions being off-limits for container placement due to safety or other prohibitive factors:

$$D = cl(D_0 \setminus \bigcup_{l=1}^{\sigma} P_l) .$$

To mathematically model the problem, the outer boundary of the region is presented as a combination of segments and circular arcs.

Each restricted area (where objects cannot be placed) is defined as a convex polygon given with vertices $q \in I_\vartheta = \{1, 2, ..., \vartheta\}$. In cases where the prohibited area is non-convex, it can be expressed as a finite union of geometric shapes such as circles and convex polygons. This allows for a more flexible representation of arbitrary shaped restricted zones within the deployment area, ensuring that all constraints related to safety or other restrictions are properly accounted for in the model.

The outlined approach for defining the area allows for an accurate approximation of any given shape. Let's assume a set of N cylindrical containers $K_i, i \in I_N$, with nuclear power plants and a site for their storage are given. An important feature of the problem is all containers being congruent. The containers should be located only on one level (placement of containers one above the other is not allowed).

The described method for defining the area enables a precise approximation of any shape. Let's consider a scenario where we have cylindrical containers $K_i, i = 1, 2, ..., N$, along with a designated storage site. A key aspect of this problem is that all containers are identical in size and shape. These containers must be placed on a single level, as stacking them on top of each other is not permitted.

The method outlined for defining the area PPP allows for the approximation of any arbitrary shape with sufficient accuracy.

Spatial form of the objects to be placed: Consider a set of cylindrical containers, $K_i, i \in I_N$, which serve as storage units for hazardous. A key characteristic of this problem is that all the containers are congruent, meaning they are uniform in shape and size. Additionally, the placement of these containers must be on a single level, with no stacking allowed, ensuring that the arrangement adheres to both technological and safety regulations crucial for preventing accidents in medical and other sensitive environments.

Given these specified features, the set of cylindrical containers can be modeled as a set of congruent cylinders K_i, $i \in I_N$, represented as a set of congruent circles C_i with radii $r, i \in I_N$. Let $u_i = (x_i, y_i)$ be the center of circle C_i. Packing of all C_i, $i \in I_N$, in $\mathbf{R}^2$ can be determined using the vector $u = (u_1, u_2, ..., u_N) \in R^{2N}$. The circle C_i translated by $u_i = (x_i, y_i)$ is denoted as $C_i(u_i)$.

The safety constraints for positioning a specific set of objects within the designated area can be categorized into two main types.

The first type of constraints is essential for maintaining necessary safety standards, particularly concerning the overall levels of contamination and compliance with the thermal storage requirements for hazardous materials. To formalize these conditions for container placement, we must evaluate the impact of each container on the site's thermal environment and contamination levels. Each container is characterized by its physical properties (such as temperature and potential for leakage), which must be considered to ensure safe storage conditions. Consequently, we assign an integral indicator that reflects the thermal and safety properties of the materials contained in each circle. This parameter will quantify the cumulative effect of each container on the temperatures of nearby containers while also influencing the overall thermal conditions and safety levels within the area.

We assume that the values of the integral coefficients k_i are determined by expert judgment k_i Then, according to the values of k_i, $i \in I_N$, we distribute the circles C_i, $i \in I_N$, into groups G_j, $j \in I_g$. Let each group consist of q_i, $i \in I_g$, circles, where g is the number of groups obtained.

In order to minimize the mutual influence of ionizing radiation and the temperature regime of the containers, we set the minimum permissible distances $d_{ij}^{g}, i, j \in I_g$, between the circles of each group G_i, $i \in I_g$, and between the circles within one group.

Thus, taking into account the first type of technological constraints will ensure an increase in fuel temperatures and the level of ionizing radiation from containers with spent nuclear fuel, and ensure a uniform distribution of ionizing radiation within the site when storing spent fuel on it.

The second type of constraints is conditioned by ensuring the conditions for servicing the containers. It is necessary to provide the possibility of approaching each container with special service equipment in order to rotate the container or move the container within the site.

To ensure this condition, it is necessary to consider the placement on the site of the so-called “service network” for moving equipment. Let’s assume that for moving equipment, it is necessary to ensure the presence of lanes with a width of d. Then, when placing objects, it is necessary to ensure the condition of touching C_i, $i \in I_N$, with a lane of a given width d, which will ensure the approach of special service equipment to the container.

The criterion for optimizing the placement of objects. As a criterion for optimization, we will choose to find the maximum filling of the selected site with circles C_i from the set I_N.

Thus, after formalizing all the conditions of the optimization problem of geometric design in an analytical form, we will formulate the problem statement as follows.

We assume that the values of the integral coefficients are established based on expert judgment. Following this, we categorize the circles into groups based on these values. Let each group contain circles, where ggg denotes the total number of groups formed.

To reduce the mutual influence of contamination and the thermal conditions of the containers, we establish minimum allowable distances dmind_{min}dmin between the circles in each group ggg and among the circles within the same group. By incorporating these safety constraints, we aim to control temperature increases and contamination levels from containers holding hazardous materials, thereby promoting a uniform distribution of safety parameters across the storage site.

The second type of constraints relates to ensuring that servicing conditions for the containers are met. It is essential to allow access to each container with specialized service equipment for the purposes of rotating or moving the containers within the site. To facilitate this, we must design a "service network" for the movement of equipment. Assuming that specific lanes of width www are necessary for this equipment, it is critical that the circles interact with the lanes of the designated width www, allowing service equipment to easily reach each container.

For the optimization criterion, we will focus on maximizing the area coverage of the selected site with circles from the set C.

With all the conditions of the geometric design optimization problem formally defined, we can proceed to articulate the problem statement as follows.

Problem Statement:

Find the vector $u = (u_1, u_2, ..., u_n) \in \mathbf{R}^{2n}$ that optimizes the placement of the maximum number of circles items from the set C_i, $i \in I_N$, within the deployment area P while ensuring safety clearence.

Mathematical formulation

One of the most essential and intricate challenges in the computer and mathematical modeling of this category of problems is the analytical representation of the interactions between circles and the designated area. In this study, we will employ the phi-function method, as outlined in previous works such as [10,11]. This approach is currently recognized as one of the most effective for addressing similar issues.

The formulation of the criteria for positioning circles within the area is grounded in the creation of the following set $G = cl(\mathbf{R}^2 \setminus P_0)$. This set can consistently be articulated as a finite union of fundamental geometric entities Q_{ij}, , expressed as follows:

$$G = \bigcup_{j=1}^{\delta} Q_{1j} \bigcup_{j=1}^{\gamma} Q_{2j} \bigcup_{j=1}^{\xi} Q_{3j} \bigcup_{j=1}^{\zeta} Q_{4j},$$

where Q_{1j} is a half-plane, Q_{2j} is a convex cone; Q_{3j} is a circle and Q_{4j} is the intersection of the half-plane and the complement of the circle to $\mathbf{R}^2$.

To address the problem, we suggest a method that simplifies the solution by transforming it into a series of problems with linear objective functions. For this purpose, the radii of circles are treated as variables. The vector of variable radii become $v^{\tau} = (r_1, r_2, ..., r_{\tau}) \in \mathbf{R}^{\tau}$.

The phi-function serve as specific measures for both the intersection of two geometric entities and the shortest distance between them, contingent upon their spatial arrangement.

By employing phi-functions for primary objects and more complex 2D shapes, the mathematical model for the series of problems can be articulated as follows:

$$F_n(X^{n*}) = \max \sum_{i=1}^{n} r_i \qquad (5)$$

where

$$X^n = (u^n, v^n) \in W_n, \quad n = 1, 2, ..., \tau + 1,$$

$$W_n = \left\{ X^n \in \mathbf{R}^{3n} : \Phi_i(X^n) \geq 0, \right.$$
$$\tilde{\Phi}_{ij}(X^n{}_j) \geq 0, i, j \in I_C,$$
$$\left. r - r_i \geq 0,\ r_i \geq 0,\ i \in I_N \right\}, \qquad (6)$$

$$\tilde{\Phi}_{ij}(X^n) = (x_i - x_j)^2 + (y_i - y_j)^2 -$$
$$-(r_i + r_j + d^g_{l_i m_j})^2,$$

$\Phi_i(X^n)$ is phi-function for C_i and $G_1 = cl(\mathbf{R}^2 \setminus P)$, I_C is a set of clusters that include C_i. And function $\Phi_i(u_i, r_i)$ describes the belonging of circle C_i to the area P.

The region of feasible solutions is defined by two categories of constraints.

The first category establishes the conditions for positioning objects within a designated feasible area while considering the specified allowable distances.

The second category delineates the conditions for placing objects at defined technological distances.

Both categories of constraints are represented using phi-functions from systems (6).

The objective function seeks to maximize the sum of the radii of the positioned circles, subject to limitations on their maximum sizes.

General solution approach

To tackle the problem, a multi-phase approach is proposed for packing containers while taking specified safety constraints into account. At each phase, nonlinear optimization techniques and contemporary NLP-solvers (Non-Linear Programing) are utilized.

This methodology centers on a multi-phase search for solutions. In order to achieve optimal utilization of space within the designated area while adhering to given constraints, the initial phase involves addressing an optimization problem aimed at maximizing the number of containers placed within a complex area that includes restricted zones. To ensure safety against elevated thermal and radiation levels, as well as to promote a uniform distribution of radiation within the area designated for storing spent nuclear fuel, minimum allowable distance constraints between groups of containers are implemented. During this phase, a modified feasible direction method utilizing an active set strategy is developed for local optimization, while a sequential statistical optimization method is created for global optimization.

In the second phase, the focus shifts to ensuring serviceability for the containers within the area by addressing the problem of arranging clusters of different geometric shapes. These clusters are required to maintain designated distances to facilitate the movement of service equipment. To tackle this issue, a nonlinear optimization approach is developed that employs an interior point method coupled with a specialized decomposition algorithm.

During the third phase, the overall safety parameters of the area are computed. If the calculated values do not meet the predefined criteria, the optimization problems from the first two phases are revisited iteratively until the desired safety conditions are achieved.

To address the problem in the first phase, a strategy has been devised that follows this sequence of methods:

The regular placements method and the block coordinate descent method are employed to establish initial points.

For searching local extrema, a modified feasible direction method integrated with an active set strategy on subdomains is utilized.

To converge towards the global extrema, a refined narrowing neighborhoods method is implemented.

The core strategy centers around optimizing the objective function defined over a set of permutations. For an effective search for global extrema, the objective function needs to be quasi-separable and exhibit multiple extrema, ensuring that the distribution of local extrema can be recognized, treating each as a realization of a random variable.

To generate initial points within the feasible domain, techniques are employed that utilize a sequence for positioning three-dimensional geometric objects. This may include methods like the block coordinate descent or the regular placements method, both aimed at arranging identical three-dimensional shapes.

Given the potential link between permutations of these geometric objects and local extrema in the problem at hand, a strategy is implemented to approximate the global extremum using a refined narrowing neighborhoods approach. This method incorporates a randomized search designed to optimize the objective function over a set of permutations. The narrowing neighborhoods technique leverages the probabilistic distribution characteristics of local extrema, facilitating the organization of sequences of three-dimensional objects in such a way that a solution nearing the global extremum can be achieved in a relatively short time frame.

To execute this method, a specific metric is established within the permutation space. The search for optimal objective function values occurs within neighborhoods defined over the permutation set. At each iteration, centers and radii for new neighborhoods are chosen based on accumulated statistical data. If there is no improvement in the objective function value during the transition to the next search phase, the radius of the neighborhood is reduced. This iterative approach leads to a convergent sequence.

It is well recognized that nonlinear optimization methods necessitate a feasible starting point. Commonly used techniques for generating initial points in three-dimensional geometric placement problems often involve various adaptations of ‘greedy’ algorithms. However, due to the NP-hard nature of geometric packing problems, relying on ‘greedy’ algorithms notably restricts the exploration of a vast array of local extrema, the number of which can exceed n!

The phi-function method facilitates the creation of a mathematical model that allows the implementation of advanced nonlinear optimization techniques throughout the problem-solving process, encompassing initial point generation, local extremum searches, and the exploration of local extrema.

In this context, a tailored approach is introduced for generating feasible points. The core concept involves enhancing the problem’s dimensionality by incorporating metric variables that are associated with the characteristics of geometric objects and their homothetic transformations.

For geometric objects amenable to homothetic transformations, we introduce variable coefficients that reflect the degree of homothety of these objects. To establish a feasible starting point, we randomly generate coordinates for the geometric objects to be placed within a designated container. Following this, we tackle a nonlinear programming problem aimed at maximizing the sum of the homothety coefficients for all geometric objects. If the optimization results in a point that corresponds to a local maximum where all homothety coefficients equal one, we accept this as the starting point for searching for local extrema in the main problem. Unlike ‘greedy’ algorithms, which may yield useful yet repetitive points, this innovative method generates diverse initial points through random coordinate selection for the centers of geometric objects.

Given the extensive set of inequalities that define the feasible region, employing nonlinear optimization methods directly to locate local extrema would incur substantial computational costs. Consequently, a specialized decomposition technique has been developed to facilitate the search for local extrema in the formulated optimization problems. This method significantly reduces computational expenses by decreasing the number of inequalities considered during the search for local extrema. By recognizing that the feasible solution space can be segmented into a union of subdomains, we can considerably lessen the time needed to identify local minima by solving a series of subproblems, each characterized by a more limited set of inequalities.

The fundamental principle of this method involves selecting a subregion within the feasible area at each stage and iteratively generating further subregions from the chosen one. Based on an analysis of the starting point, an additional set of constraints is established for the placement parameters of each object, allowing for movement within individual containers. Subsequently, inequalities are eliminated for all pairs of three-dimensional geometric objects whose respective containers do not intersect. This reduction in constraints also decreases the number of additional variables when dealing with quasi-phi functions. Following this, a search for a local minimum is conducted for the newly constructed subproblem. The local extremum obtained from this subproblem then serves as the starting point for the next iteration.

Searching for local optimum solutions

To identify local maxima in the given nonlinear programming problem, we applied an optimization method based on the feasible direction strategy. This approach focuses on moving within the feasible region of the solution space while ensuring that each step leads toward an increase in the objective function, adhering to the problem's constraints.

Based on the properties of mathematical model, a subregion $W_{k_1} \subset W$ containing the obtained initial point $X^i \in W$ is selected to find the corresponding local minimum X^{1*}. The local minimum is then computed within this subregion. If there are other subregions $W_{k_j} \subset W$, where $X^{1*} \in W_{k_j}$, $j = 1, 2, ..., \vartheta$, the initial point lies, and the computed solution X^{1*} is not a local minimum, the search for a local minimum in these subregions is repeated. This process continues iteratively until a local minimum of the overall problem is found. Thus, the task of finding a local minimum is reduced to solving a sequence of nonlinear programming problems

$$F(X^{j*}) = \min_{X \in W_{k_j}} F(X),\ j = 1, 2, ..., m \square\ \eta_0.$$

To address the problem, a modified version of Zoutendijk's feasible direction method is applied, along with the ε-active inequality strategy. For each subproblem, a standard iterative procedure $X^{(k+1)} = X^k + tZ^k$, $k = 1, 2, ..., \zeta$, is used to determine the local minimum. In each iteration, the process involves solving a linear programming problem, where the solution is derived from the following linear programming formulation:

$$\max_{(\alpha^k, Z^k) \in G^k} \alpha^k,$$

$$G^k = \{(\alpha^k, Z^k) \in \mathbf{R}^{3n+1} :$$

$$(-\nabla F(X^k), Z^k) \geq \alpha^k,$$

$$(-\nabla \Psi_{k_j}(X^k), Z^k) \geq \alpha^k,$$

$$j = 1, 2, ..., \varsigma_k(\varepsilon_k),$$

$$\left| z_i^k \right| \leq 1,\ i = 1, 2, ..., 3n+1\}$$

where $\Psi_{k_j}(X^k)$ is left parts of ε-active inequalities from the system separated from the system at a point X^k.

The transition from one problem to the next is performed as follows.

Let $X^i = \bar{X}^1 \in W$ be a starting point. Then from the system specifying the subregion W_{k_1}, a system $W_{k_1} \subset W$, such that $\bar{X}^1 \in W_{i_1}$ is chosen. Using the point $\bar{X}^1$ as a starting point, the problem

$$F(X^{1*}) = \min_{X \in W_{k_1}} F(X)$$

is solved.

A solution X^{1*} can be either a local minimum over the entire region W or geometric objects relative to the subregion W_{i_1}.

In order to determine whether X^{1*} a local minimum relative to W, it is necessary to investigate subregions W_{k_j} with $X^{1*} \in W_{k_j}$, $j \in \{1, 2, ..., \eta_0\}$. For this purpose, all are chosen from the system ε-active inequalities in X^{1*} and problem is solved.

After that, a new system of inequalities is formed that defines the subdomain $W_{k_2} \subset W$, such that $\bar{X}^2 \in W_{k_2}$. Using $\bar{X}^2$ as a starting point, the problem

$$F(X^{2*}) = \min_{X \in W_{k_2}} F(X)$$

is solved.

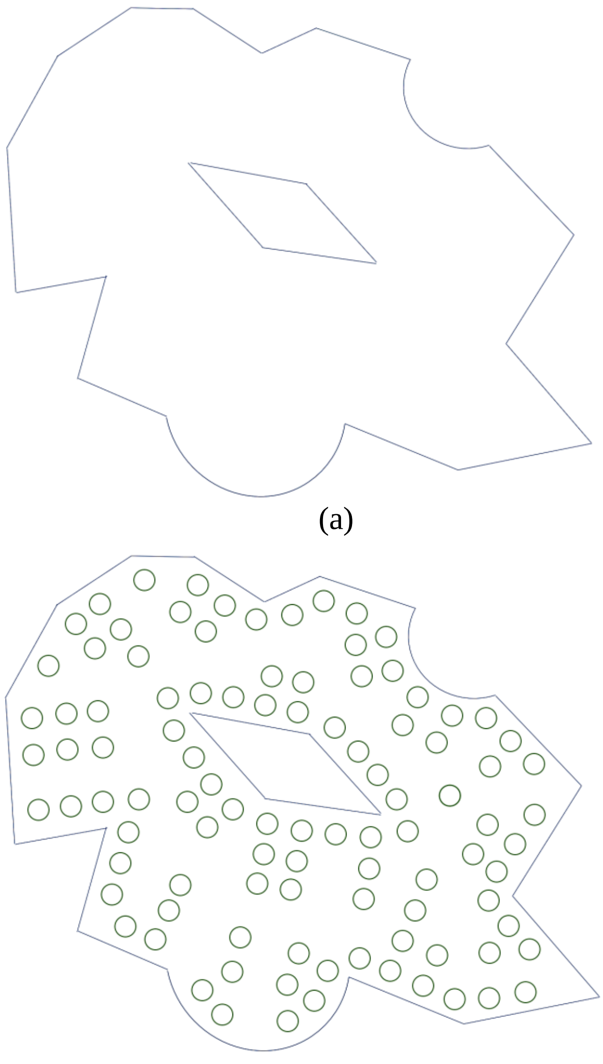
The described process continues until a local minimum of the underlying problem is reached. At this point, no further feasible steps can improve the objective function, signaling the optimization's convergence to a local solution.

Computational results

To evaluate the performance of our constructed mathematical model and the proposed approach for solving the problem, we tackled the challenge of packing 80 cylindrical containers of hazardous waste within an area of complex geometry (depicted in Figure 1,a). The geometric contours of this area are defined by a sequence of line segments and circular arcs. Within the designated placement area, there is a zone where container placement is prohibited. Additionally, the problem statement specifies technological constraints on container placement. The results of solving this problem are illustrated in Figure 1,b.

Solving the problem shown in the figure took 45 minutes and 40 seconds. We used a computer with an Intel(R) Core i5-10400F 2.90GHz processor and 16 GB of RAM, with our software developed in C#.

Recent advances in optimization methods, especially in nonlinear optimization, have revolutionized the way we solve these problems. These improvements have greatly boosted the reliability, speed, and accuracy in finding both local and global solutions. They can be applied across various domains, utilizing user-developed external procedures for calculating objective functions, residual constraints, and Jacobian and Hessian matrices.



(a)

(b)

Fig. 1. A numerical example

In this study, we utilized the IPOPT library to enhance the efficiency of locating local extrema within subregions. IPOPT is particularly well-suited for large-scale packing optimization problems due to its ability to manage high-dimensional challenges effectively. Such problems often involve numerous variables and constraints, which become increasingly difficult for traditional optimization methods to solve within reasonable time limits as the problem size grows. IPOPT's advanced interior-point algorithms allow it to navigate these complex solution spaces with speed and precision.

By exploiting the structural properties of optimization problems, IPOPT efficiently searches for optimal solutions while ensuring all constraints are satisfied. It leverages sparse linear algebra and problem-specific features, enabling it to scale to problems with millions of variables. This makes IPOPT a vital tool for handling large-scale packing optimization tasks. Additionally, IPOPT offers extensive customization options and interfaces, allowing users to tailor the optimization process to meet specific needs and integrate it seamlessly into existing workflows.

Conclusion

This study formulates the problem of optimally arranging containers for hazardous materials, considering specified sanitary constraints, as a geometric design optimization issue. All aspects of this geometric design problem are elaborately detailed. A mathematical model is developed for packing congruent circles into a multiply connected region, where the boundary consists of circular arcs and line segments. This model is framed as a nonlinear optimization problem.

Utilizing the method of phi-functions, we construct a mathematical model where the feasible solution space can be represented as a union of subregions. Each subregion is defined by systems of inequalities with continuous functions on the left-hand sides. This representation facilitates the effective application of modern nonlinear optimization techniques to address the problem.

The findings of this research carry significant practical implications, especially regarding the optimization of safety-critical systems. The intelligent system developed for the optimal placement of containers is directly applicable to the secure storage of hazardous materials. By employing advanced mathematical modeling techniques, this approach navigates complex real-world constraints, including spatial limitations, safety regulations, and technological requirements.

The effectiveness of the proposed model is illustrated through its application to a practical scenario: optimally packing 80 cylindrical containers within a complex, multiconnected storage area featuring prohibited zones. This scenario reflects real-world challenges, highlighting the system's ability to enhance safety and efficiency in storage operations across various industries.

The practical benefits extend beyond theoretical insights, providing tangible improvements in operational settings. The system's capability to handle nonlinear programming challenges with high precision ensures effective integration into existing safety protocols. This integration can lead to optimized storage density, reduced risk of accidents, and improved utilization of available space — all critical factors in environments where safety is a priority.

Furthermore, the flexibility of this approach allows for adaptation across a wide array of applications, ranging from industrial storage facilities to the transportation of hazardous materials. As safety regulations become increasingly stringent, the demand for such intelligent systems will rise, rendering the results of this study not only relevant but essential for future advancements in safety optimization.

In summary, the practical significance of the results obtained lies in providing a robust, efficient, and scalable solution to the pressing issue of safe storage of hazardous materials, with potential applications across multiple domains where safety and optimization are vital.

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