МІНІСТЕРСТВО ОСВІТИ І НАУКИ, МОЛОДІ ТА СПОРТУ УКРАЇНИ
ХАРКІВСЬКИЙ НАЦІОНАЛЬНИЙ ЕКОНОМІЧНИЙ УНІВЕРСИТЕТ

Методичні рекомендації до виконання практичних завдань з навчальної дисципліни «Вища та прикладна математика» для студентів-іноземців та студентів, що навчаються іноземною мовою, напряму підготовки 6.140103 «Туризм» денної форми навчання

Харків. Вид. ХНЕУ, 2012

Ministry of Education and Science, Youth and Sports of Ukraine
KHARKIV NATIONAL UNIVERSITY OF ECONOMICS

Guidelines
for practical tasks of the educational discipline “Higher and applied mathematics” for foreign and English-learning full-time students of the direction 6.140103 “Tourism”

Compiled by Ie. Misiura

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Харків. Вид. ХНЕУ, 2012

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Guidelines for practical tasks of the educational discipline “Higher and applied mathematics” for foreign and English-learning full-time students of the direction 6.140103 “Tourism” / compiled by Ie. Misiura. – Kh. : KNUE, 2012. – 36 p. (English, Ukrainian)

Methodical recommendations are intended for foreign and English-learning students of the direction 6.140103 "Tourism" for practical studies of “Random variables and their characteristics” of the discipline "Higher and applied mathematics".

The sufficient theoretical material and the number of solved typical examples of each theme give students the possibility to master “Random variables and their characteristics” and apply the obtained knowledge in practice on their own.

Practical tasks for self-work and the list of theoretical questions which promote improving and extending students’ knowledge of all the themes are given.

It is recommended for foreign and English-learning full-time students of the direction "Tourism".

Подано методичні рекомендації для студентів-іноземців та студентів, що навчаються англійською мовою, напряму підготовки 6.140103 «Туризм» денної форми навчання для практичних занять з теми «Випадкові величини та їх характеристики» навчальної дисципліни «Вища та прикладна математика».

Викладено необхідний теоретичний матеріал та наведено типові приклади, які допоможуть найбільш повному засвоєнню матеріалу з теми «Випадкові величини та їх характеристики» та застосуванню отриманих знань на практиці.

Подано завдання для самостійної роботи та перелік теоретичних питань, що сприяють удосконаленню та поглибленню знань студентів з даної теми.

Рекомендовано для студентів-іноземців та студентів, що навчаються англійською мовою, напряму підготовки «Туризм» денної форми навчання.

Introduction

Methodical recommendations are intended for foreign and English-learning students of the direction 6.140103 "Tourism" for practical studies of “Random variables and their characteristics” of the discipline "Higher and applied mathematics".

Its aim is the practical application of mathematic apparatus to the problem solutions in “Random variables and their characteristics”, which consists of the following themes: discrete random variables, their distribution laws and numerical characteristics; continuous random variables, a function and a density of a distribution of probabilities, numerical characteristics; a uniform, an exponential and a normal laws of probabilities distribution.

The sufficient number of solved typical examples of each theme gives students the possibility to master “Random variables and their characteristics” and apply the obtained knowledge in practice on their own. At the end of methodical recommendations there are tasks for self-work and the list of theoretical questions which promote improving and extending students’ knowledge of all the themes.

1. Discrete random variables

A variable is called *random,* if it can receive real values with definite probabilities as a result of experiment.

The random variable X is called *discrete*, if such non-negative function exists

$$P(X = x_i) = p_i,\ i = 1,2,\dots,n,\ \sum_{i=1}^{n} p_i = 1,$$

which determines the correspondence between a value x_i of the variable X and the probability p_i, that X receives this value.

A random variable is denoted by X, Y, Z and so on and its possible values are denoted by x_i, y_i, z_i For example, if X is a random variable, then its values are $x_1, x_2, \dots, x_n$ (these values form a complete group of events, therefore $\sum_{i=1}^{n} p_i = 1$).

Distribution law (row) of a discrete random variable is called a set of all its possible values and probabilities which these values are possessed. It's often written in the form of a table

x_i	x_1	x_2	$\cdots$	x_n
p_i	p_1	p_2	$\cdots$	p_n

Example 1.1. Two balls are drawn in succession without replacement from an urn containing 4 red balls and 3 black balls. The possible outcomes and the values x_i of the random variable X, where X is the number of red balls, are

Sample space	x_i
RR	2
RB	1
BR	1
BB	0

Let's find the probability of each value x_i:

$$P(X=0)=P(BB)=\frac{1}{4},$$

$$P(X=1)=P(RB+BR)=\frac{1}{4}+\frac{1}{4}=\frac{1}{2},$$

$$P(X=2)=P(RR)=\frac{1}{4}.$$

Let's write the distribution law of this discrete random variable:

x_i	0	1	2
p_i	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

Example 1.2. Letting X denote the random variable that is defined as

the sum of two fair dice, then

$$P(X=2)=P((1,1))=\frac{1}{36},$$

$$P(X=3)=P((1,2),(2,1))=\frac{2}{36},$$

$$P(X=4)=P((1,3),(2,2),(3,1))=\frac{3}{36},$$

$$P(X=5)=P((1,4),(2,3),(3,2),(4,1))=\frac{4}{36},$$

$$P(X=6)=P((1,5),(2,4),(3,3),(4,2),(5,1))=\frac{5}{36},$$

$$P(X=7)=P((1,6),(2,5),(3,4),(4,3),(5,2),(6,1))=\frac{6}{36},$$

$$P(X=8)=P((2,6),(3,5),(4,4),(5,3),(6,2))=\frac{5}{36},$$

$$P(X=9)=P((3,6),(4,5),(5,4),(6,3))=\frac{4}{36},$$

$$P(X=10)=P((4,6),(5,5),(6,4))=\frac{3}{36},$$

$$P(X=11)=P((5,6),(6,5))=\frac{2}{36},$$

$$P(X=12)=P((6,6))=\frac{1}{36}.$$

Let’s write the distribution law of this discrete random variable:

x_i	2	3	4	5	6	7	8	9	10	11	12
p_i	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

Distribution law (row) can be graphically plotted (Fig. 1.1). Values of a variable x_i are marked on X-axis, the corresponding probabilities p_i are marked on Y-axis. The obtained points are connected with the help of segments. It results *a distribution polygon*.

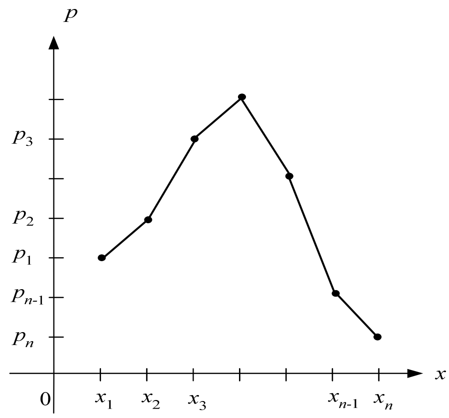

Fig. 1.1. **A distribution polygon**

Example 1.3. Distribution law of a discrete random variable X

x_i	-2	2	6	10	14
p_i	0.05	0.16	0.35	0.31	0.13

is given. Draw a distribution polygon.

Solution. Let's plot a distribution polygon for the given distribution law (Fig. 1.2).

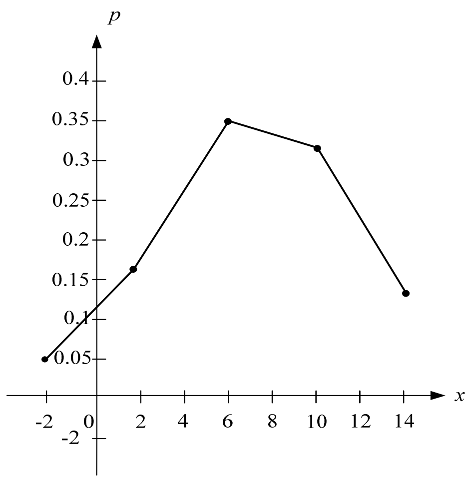

Fig. 1.2. **A distribution polygon**

1.1. Numerical characteristics of discrete random variables

Mathematical expectation of a discrete random variable X is called a sum of products of possible values x_i, which a variable X is taken, and their corresponding probabilities p_i.

$$M(X)=\sum_{i=1}^{n} x_i \cdot p_i = x_1 \cdot p_1 + x_2 \cdot p_2 + ... + x_n \cdot p_n .$$

Properties of mathematical expectation:

1) $M(cX)=c \cdot M(X)$, $c \in R$;

2) $M(X+Y)=M(X)+M(Y)$, where X, Y are the discrete random variables;

3) $M(X \cdot Y)=M(X) \cdot M(Y)$ for independent random variables X and Y.

Discrete random variables X and Y are called *independent random variables*, if the events $X = x_i$ and $Y = y_i$ are independent for arbitrary i and j.

Variance of a random variable X is called a mathematical expectation of deviation square of a random variable from its mathematical expectation, i.e.

$$D(X)=M\left[(X-M(X))^2\right]=\sum_{i=1}^{n}(x_i - M(X))^2 \cdot p_i \text{ or}$$

variance of a random variable X equals a mathematical expectation of its square minus a square of its mathematical expectation, i.e.

$$D(X)=M(X^2)-[M(X)]^2,$$

where $M(X^2)=\sum_{i=1}^{n} {x_i}^2 \cdot p_i$.

Properties of variance:

1) $D(cX)=c^2 \cdot D(X)$, $c \in R$;

2) $D(X+Y)=D(X)+D(Y)$ for independent random variables X and Y.

Root-mean-square deviation (or standard deviation) of a random variable X is the square root of its variance, i.e.

$$\sigma(X)=\sqrt{D(X)}.$$

Example 1.4. Distribution law of a discrete random variable X

x_i	-2	2	6	10	14
p_i	0.05	0.16	0.35	0.31	0.13

is given. Find a mathematical expectation $M(X)$, a variance $D(X)$ and root-mean-square deviation $\sigma(X)$ of a discrete random variable X.

Solution. Let's find the numerical characteristics of a random variable:

a) a mathematical expectation is defined by the following formula:

$$M(X) = \sum_{i=1}^{5} x_i \cdot p_i = x_1 \cdot p_1 + x_2 \cdot p_2 + x_3 \cdot p_3 + x_4 \cdot p_4 + x_5 \cdot p_5 =$$

$$= -2 \cdot 0.05 + 2 \cdot 0.16 + 6 \cdot 0.35 + 10 \cdot 0.31 + 14 \cdot 0.13 =$$

$$= -0.1 + 0.32 + 2.1 + 3.1 + 1.82 = 7.24;$$

b) a variance is defined by the following formulas:

1) $D(X) = \sum_{i=1}^{5} (x_i - M(X))^2 \cdot p_i = (-2 - 7.24)^2 \cdot 0.05 + (2 - 7.24)^2 \cdot 0.16 +$

$$+ (6 - 7.24)^2 \cdot 0.35 + (10 - 7.24)^2 \cdot 0.31 + (14 - 7.24)^2 \cdot 0.13 = 17.5024;$$

2) $D(X) = M(X^2) - [M(X)]^2$, where $M(X^2) = \sum_{i=1}^{5} x_i^2 \cdot p_i$.

$$M(X^2) = x_1^2 \cdot p_1 + x_2^2 \cdot p_2 + x_3^2 \cdot p_3 + x_4^2 \cdot p_4 + x_5^2 \cdot p_5 =$$

$$= (-2)^2 \cdot 0.05 + 2^2 \cdot 0.16 + 6^2 \cdot 0.3 + 10^2 \cdot 0.31 + +14^2 \cdot 0.13 = 69.92,$$

$$D(X) = 69.92 - 7.24^2 = 17.5024;$$

c) a root-mean-square deviation is defined by the following formula:

$$\sigma(X) = \sqrt{D(X)} = \sqrt{17.5024} = 4.1836.$$

The probability of the fact that a random variable X receives a value less than x, is called a *distribution function* of a random variable X and marked as $F(x)$:

$$F(x) = P(X \le x).$$

General properties of the cumulative distribution function:

1. $F(x)$ is a bounded function, i.e. $0 \le F(x) \le 1$.

2. $F(x)$ is a non-decreasing function for $x \in (-\infty, \infty)$, i.e. if $x_2 > x_1$, then $F(x_2) \ge F(x_1)$.

3. $\lim\limits_{x \to -\infty} F(x) = F(-\infty) = 0$.

4. $\lim\limits_{x \to +\infty} F(x) = F(+\infty) = 1$.

5. The probability that a random variable X lies in the interval (x_1, x_2) is equal to the increment of its cumulative distribution function on this interval, i.e.

$$P(x_1 < X < x_2) = F(x_2) - F(x_1).$$

6. $F(x)$ is left continuous; i.e. $\lim\limits_{x \to x_0 - 0} F(x) = F(x_0)$.

Example 1.5. Find a distribution function of a random variable X, which is defined by a distribution law

x_i	1	2	3	4
p_i	0.7	0.21	0.063	0.027

Find the probability that a random variable X possesses a value less than 1 and more than 4.

Solution. A random variable X doesn't possess the values less than 1, thus for $x \le 1$ events $X < x$ are impossible and $F(x) = 0$.

If $1 < x \le 2$, then $F(x) = 0.7$, because X can possess only the value $x = 1$ with the probability $p = 0.7$.

If $2 < x \le 3$, then $F(x) = 0.7 + 0.21 = 0.91$, because X can possess only of values $x = 1$or $x = 2$ with the probability $p = 0.7$ and $p = 0.21$ (addition theorem for independent events).

If $3 < x \le 4$, then $F(x) = 0.7 + 0.21 + 0.063 = 0.973$, because X can possess only of values $x = 1$, $x = 2$ or $x = 3$ with the probability $p = 0.7$, $p = 0.21$ and $p = 0.063$ (addition theorem for independent events).

If $x > 4$, then $F(x) = 1$, because the event $X \le 4$ is reliable and its probability equals 1.

The required integral function is defined by the formula:

$$F(x)=\begin{cases}0, & if\ x\le 1\\ 0.7, & if\ 1<x\le 2\\ 0.91, & if\ 2<x\le 3\\ 0.973, & if\ 3<x\le 4\\ 1, & if\ x>4\end{cases}.$$

A graph of integral function will be of the form:

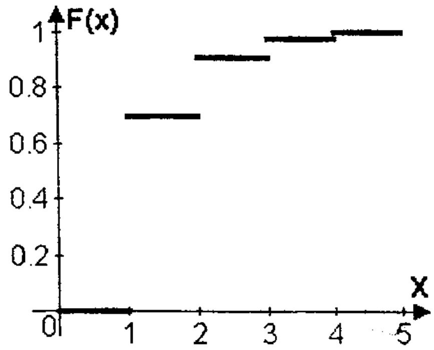

Fig. 1.3. **A graph of integral function**

Let's find the probability that a random variable X possesses a value less than 1 and more than 4, i.e. $P(1<X<4)$:

$$P(1<X<4)=F(4)-F(1)=0.973-0=0.973.$$

1.2. Main discrete distributions

1.2.1. Binomial distribution law. A random variable X has the *binomial distribution* with parameters (n,p) (see Fig. 1.5) if

$$P_n(x=k)=C_n^k p^k q^{n-k},\ k=0,1,...,n,$$

where $0<p<1,\ q=1-p,\ n\ge 1$.

The numerical characteristics are given by the formulas

$$M(X)=np, \qquad D(X)=npq, \qquad \sigma(X)=\sqrt{npq}.$$

Binomial distribution law of a discrete random variable is often written in the form of a table:

x_i	0	1	…	$n-1$	n
p_i	q^n	$C_n^1 pq^{n-1}$	…	$C_n^{n-1} p^{n-1} q$	p^n

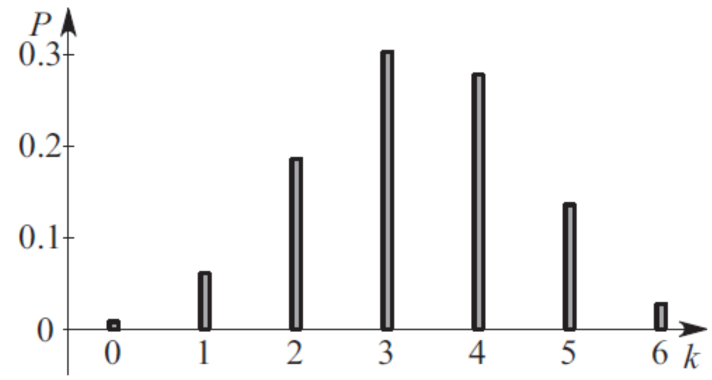

Fig. 1.5. **Binomial distribution law for** $p = 0.55$**,** $n = 6$

The binomial distribution is a model of random experiments consisting of n independent identical Bernoulli trials.

Example 1.6. The probability of passing an exam excellently for each of three students equals 0.4. Make up a distribution law of a number of excellent marks which are got by the students at the exam. Find a mathematical expectation, a variance and a root-mean square deviation of a discrete random variable.

Solution. Let a discrete random variable X be a number of students with the mark "5". It has such possible values:

$x_1 = 0$ (no student passed the exam with the mark "5");

$x_2 = 1$ (one student passed the exam with the mark "5");

$x_3 = 2$ (two students passed the exam with the mark "5");

$x_4 = 3$ (three students passed the exam with the mark "5").

Students' passing an exam with the mark "5" are independent events. The probabilities of passing an exam of each student are equal, then we use Bernoulli's formula:

$$P_n(x = k) = C_n^k p^k q^{n-k}.$$

According to the condition we have: $n = 3$, $p = 0.4$, $q = 1 - p = 0.6$.

Let's find

$x_1 = 0$, $P_3(0) = C_3^0 p^0 q^{3-0} = 1 \cdot 1 \cdot q^3 = 0.6^3 = 0.216$;

$x_2 = 1$, $P_3(1) = C_3^1 p^1 q^{3-1} = 3 \cdot p \cdot q^2 = 3 \cdot 0.4 \cdot 0.6^2 = 0.432$;

$x_3 = 2$, $P_3(2) = C_3^2 p^2 q^{3-2} = 3 \cdot p^2 \cdot q = 3 \cdot 0.4^2 \cdot 0.6 = 0.288$;

$x_4 = 3$, $P_3(3) = C_3^3 p^3 q^{3-3} = 1 \cdot p^3 \cdot q^0 = 1 \cdot 0.4^3 \cdot 1 = 0.064$.

A distribution law of a discrete random variable X is defined by the table:

x_i	0	1	2	3
p_i	0.216	0.432	0.288	0.064

According to the formulas for a mathematical expectation, a variance and a root-mean square deviation we obtain:

$M(X) = np = 3 \cdot 0.4 = 1.2$;

$D(X) = npq = 3 \cdot 0.4 \cdot 0.6 = 0.72$;

$\sigma(X) = \sqrt{npq} = \sqrt{0.72} \approx 0.85$.

1.2.2. Geometric distribution law. A random variable X has a *geometric distribution* with parameters p (see Fig. 1.6) if

$$P_n(x = k) = pq^k, \; k = 0, 1, 2, \ldots$$

where $0 < p < 1$, $q = 1 - p$, $n \geq 1$.

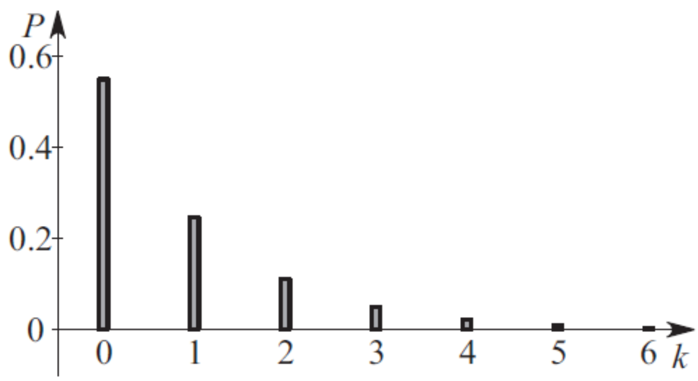

Fig. 1.6. Geometric distribution law for $p = 0.55$, $n = 6$

The numerical characteristics can be calculated by the formulas

$$M(X)=\frac{1-p}{p}=\frac{q}{p}, \qquad D(X)=\frac{1-p}{p^2}=\frac{q}{p^2}, \qquad \sigma(X)=\sqrt{\frac{q}{p^2}}.$$

The geometric distribution describes a random variable X equal to the number of failures before the first success in a sequence of Bernoulli trials with probability p of success in each trial.

1.2.3. Hypergeometric distribution law. A random variable X has the *hypergeometric distribution* with parameters (N, p, n) (see Fig. 1.7) if

$$P_n(x=k)=\frac{C_{Np}^k\, C_{Nq}^{n-k}}{C_N^n},\ k=0,1,\ldots,n,$$

where $0<p<1$, $q=1-p$, $0\le n\le N$, $N>0$.

If $n<<N$ (in practice, $n<0{,}1N$), then

$$\frac{C_{Np}^k\, C_{Nq}^{n-k}}{C_N^n}\approx C_n^k p^k q^{n-k},$$

i.e., the hypergeometric distribution tends to the binomial distribution.

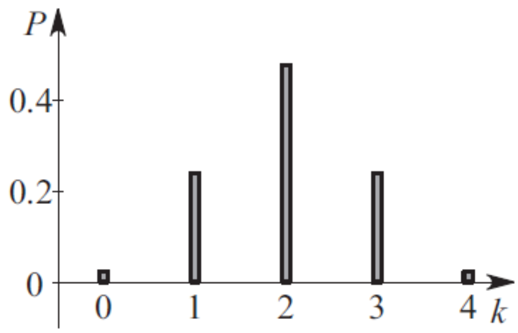

Fig. 1.7. **Hypergeometric distribution law for** $p=0.5$**,** $N=10$, $n=4$

The numerical characteristics are given by the formulas

$$M(X)=np, \qquad D(X)=\frac{N-n}{N-1}npq, \qquad \sigma(X)=\sqrt{\frac{N-n}{N-1}npq}.$$

A typical scheme in which the hypergeometric distribution arises is as follows: n elements are randomly drawn without replacement from a population of N elements containing exactly Np elements of type *I* and Nq elements of type *II*. The number of elements of type *I* in the sample is described by the hypergeometric distribution.

1.2.4. Poisson distribution law. A random variable X has the *Poisson distribution* with parameters λ $(\lambda > 0)$ (see Fig. 1.8) if

$$P_n(x=k) = \frac{\lambda^k e^{-\lambda}}{k!}, \ k = 0, 1, 2, \ldots,$$

where $\lambda = np$, $k! = 1 \cdot 2 \cdot \ldots \cdot k$.

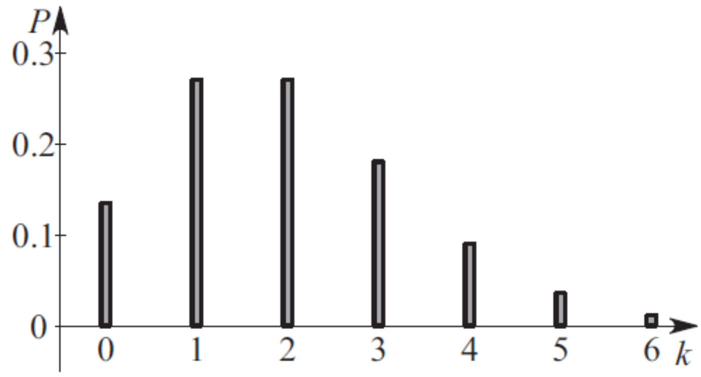

Fig. 1.8. **Poisson distribution law for** $\lambda = 2$

Poisson distribution law with parameter λ of a discrete random variable is often written in the form of a table:

x_i	0	1	2	…	n
p_i	$e^{-\lambda}$	$\lambda e^{-\lambda}$	$\frac{\lambda^2 e^{-\lambda}}{2!}$	…	$\frac{\lambda^n e^{-\lambda}}{n!}$

Poisson distribution can be obtained as a limit of a binomial distribution if n goes to ∞ $(n \to \infty)$ and p goes to 0 $(p \to 0)$ (in this case $q = 1 - p \to 1$).

The numerical characteristics can be calculated by the formulas

$$M(X) = np = \lambda; \qquad D(X) = npq \approx \lambda; \qquad \sigma(X) = \sqrt{npq} = \sqrt{\lambda}.$$

The Poisson distribution is the limit distribution for many discrete distributions such as the hypergeometric distribution, the binomial distribution, distributions arising in problems of arrangement of particles in cells, etc. The Poisson distribution is an acceptable model for describing the random number of occurrences of certain events on a given time interval in a given domain in space.

Example 1.7. The probability of finding a mistake on a book page is equal to 0.004. 500 pages are checked. Make up a distribution law of a number of finding a mistake on a book page. Find a mathematical expectation, a variance and a root-mean square deviation of a discrete random variable.

Solution. Let X be a number of finding a mistake on a book page, then the possible values of X are 0, 1, 2, 3, …, 500. Here $p = 0.004, n = 500$, then $\lambda = 500 \cdot 0.004 = 2$. Let's make up a distribution law of X according to Poisson formula $P_n(x = k) = \frac{\lambda^k e^{-\lambda}}{k!} = \frac{2^k e^{-2}}{k!}$:

$P_{500}(x = 0) = \frac{2^0 \cdot e^{-2}}{0!} \approx 0.13534$, $P_{500}(x = 1) = \frac{2^1 \cdot e^{-2}}{1!} \approx 0.27067$,

$P_{500}(x = 2) = \frac{2^2 \cdot e^{-2}}{2!} \approx 0.27067$, $P_{500}(x = 3) = \frac{2^3 \cdot e^{-2}}{3!} \approx 0.18045$,

$P_{500}(x = 4) = \frac{2^4 \cdot e^{-2}}{4!} \approx 0.09022$, $P_{500}(x = 5) = \frac{2^5 \cdot e^{-2}}{5!} \approx 0.03609$,

$P_{500}(x = 6) = \frac{2^6 \cdot e^{-2}}{6!} \approx 0.01203$, $P_{500}(x = 7) = \frac{2^7 \cdot e^{-2}}{7!} \approx 0.00344$,

$P_{500}(x = 8) = \frac{2^8 \cdot e^{-2}}{8!} \approx 0.00086$, $P_{500}(x = 9) = \frac{2^9 \cdot e^{-2}}{9!} \approx 0.00019$,

$P_{500}(x = 10) = \frac{2^{10} \cdot e^{-2}}{10!} \approx 0.00004$ and so on. At $k \ge 11$ we have that $P_{500}(x \ge 11) \approx 0$.

So, a distribution law has a form

x_i	0	1	2	3	4	5
p_i	0.13534	0.27067	0.27067	0.18045	0.09022	0.03609

x_i	6	7	8	9	10	…
p_i	0.01203	0.00344	0.00086	0.00019	0.00004	…

Let’s calculate the total sum of probabilities:

$$\sum_{i=1}^{n} p_i = \sum_{i=1}^{500} p_i \approx 0.99999 \approx 1.$$

Let’s find the numerical characteristics:

$$M(X) = np = \lambda = 2; \qquad D(X) = npq \approx \lambda = 2;$$

$$\sigma(X) = \sqrt{npq} = \sqrt{\lambda} = \sqrt{2} \approx 0.41421.$$

2. Continuous random variables

A *continuous random variable* is a random variable where the data can take infinitely many values on some numerical interval or a random variable which takes an infinite number of possible values. Continuous random variables are usually measurements.

Examples include height, weight, the amount of sugar in an orange, the time required to run a mile.

A continuous random variable is characterized by two functions: a density function (the differential distribution function) $f(x)$ and a distribution function (the integral distribution function) $F(x)$.

The random variable X is called a *continuous random variable*, if for any numbers $a < b$ such non-negative function $f(x)$ exists, that

$$P(a < X < b) = \int_{a}^{b} f(x)dx.$$

The function $f(x)$ is called a *density function* of continuous random variable.

General properties of the density function:

1. $f(x)$ is a non-negative function, i.e. $f(x) \geq 0$ for all *x*.
2. $f(x)$ is a non-decreasing function for $x \in (-\infty, \infty)$, then

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

(the condition of normalization of the function $f(x)$).

3. The relationship between the functions $f(x)$ and $F(x)$:

$$F(x) = \int_{-\infty}^{x} f(x)dx.$$

The probability of the fact that a random variable X receives a value less than x, is called a *cumulative distribution function* of random variable X and marked as $F(x)$:

$$F(x) = P(X \le x).$$

General properties of the integral distribution function:

1. $F(x)$ is a bounded function, i.e. $0 \le F(x) \le 1$.

2. $F(x)$ is a non-decreasing function for $x \in (-\infty, \infty)$, i.e. if $x_2 > x_1$, then $F(x_2) \ge F(x_1)$.

3. $\lim\limits_{x \to -\infty} F(x) = F(-\infty) = 0$.

4. $\lim\limits_{x \to +\infty} F(x) = F(+\infty) = 1$.

5. The relationship between the functions $f(x)$ and $F(x)$: $f(x) = F'(x)$.

6. The probability that a random variable X lies in the interval (x_1, x_2) is equal to the increment of its cumulative distribution function on this interval; i.e.

$$P(x_1 < X < x_2) = F(x_2) - F(x_1) = \int_{x_1}^{x_2} f(x)dx.$$

Example 2.1. The density function of a continuous variable X is given:

$$f(x) = \begin{cases} 0, & x \le 0 \\ c\left(4x - 2x^2\right), & 0 < x \le 2. \\ 0, & x > 2 \end{cases}$$

a) What is the value of c? b) Find $P(X > 1)$.

Solution. Since $f(x)$ is a probability density function, we must have the condition of normalization of this function that $\int_{-\infty}^{\infty} f(x)dx = 1$, implying that

$$\int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{0} 0dx + \int_{0}^{2} c\left(4x - 2x^2\right)dx + \int_{2}^{+\infty} 0dx = 1$$

or

$$\int_{0}^{2} c\left(4x - 2x^2\right)dx = 1$$

or

$$c\left(4\frac{x^2}{2} - 2\frac{x^3}{3}\right)\Bigg|_0^2 = c\left(2x^2 - \frac{2x^3}{3}\right)\Bigg|_0^2 = 1$$

or

$$c\left(8 - \frac{16}{3} - 0\right) = c \cdot \frac{8}{3} = 1$$

or

$$c = \frac{3}{8}.$$

So, a probability density function is

$$f(x) = \begin{cases} 0, & x \le 0 \\ \frac{3}{8}\left(4x - 2x^2\right), & 0 < x \le 2 \\ 0, & x > 2 \end{cases}$$

Let's find $P(X > 1)$:

$$P(X > 1) = P(1 < X < +\infty) = \int_{1}^{+\infty} f(x)dx = \int_{1}^{2} \frac{3}{8}\left(4x - 2x^2\right)dx + \int_{2}^{+\infty} 0dx =$$

$$= \frac{3}{8}\left(2x^2 - \frac{2x^3}{3}\right)\Bigg|_1^2 = \frac{3}{8}\left(\left(2 \cdot 2^2 - \frac{2 \cdot 2^3}{3}\right) - \left(2 \cdot 1^2 - \frac{2 \cdot 1^3}{3}\right)\right) = \frac{3}{8}\left(\frac{8}{3} - \frac{4}{3}\right) = \frac{1}{2}.$$

Example 2.2. The differential distribution function of a continuous variable X is given:

$$f(x)=\begin{cases}0, & x\le 1\\ x-\dfrac{1}{2}, & 1<x\le 2.\\ 0, & x>2\end{cases}$$

Find the integral distribution function $F(x)$. Plot the graphs of the functions $f(x)$ and $F(x)$.

Solution. The integral distribution function $F(x)$ according to the formula is $F(x)=\int\limits_{-\infty}^{x} f(x)dx$.

If $x\le 1$, then $f(x)=0$ and $F(x)=\int\limits_{-\infty}^{x} f(x)dx=\int\limits_{-\infty}^{x} 0dx=0$.

If $1<x\le 2$, then $F(x)=\int\limits_{-\infty}^{x} f(x)dx=\int\limits_{-\infty}^{1} f(x)dx+\int\limits_{1}^{x} f(x)dx=$

$$=\int\limits_{-\infty}^{1} 0dx+\int\limits_{1}^{x}\left(x-\frac{1}{2}\right)dx=\left(\frac{x^2}{2}-\frac{1}{2}x\right)\Bigg|_1^x=\frac{x^2}{2}-\frac{1}{2}x-\left(\frac{1}{2}-\frac{1}{2}\right)=\frac{x^2}{2}-\frac{x}{2}.$$

If $x>2$, then $F(x)=\int\limits_{-\infty}^{x} f(x)dx=\int\limits_{-\infty}^{1} f(x)dx+\int\limits_{1}^{2} f(x)dx+\int\limits_{2}^{x} f(x)dx=$

$$=\int\limits_{-\infty}^{1} 0dx+\int\limits_{1}^{2}\left(x-\frac{1}{2}\right)dx+\int\limits_{2}^{x} 0dx=\left(\frac{x^2}{2}-\frac{1}{2}x\right)\Bigg|_1^2=\frac{2^2}{2}-\frac{1}{2}\cdot 2-\left(\frac{1^2}{2}-\frac{1}{2}\cdot 1\right)=$$
$$=2-1-0=1.$$

Let's write the formula for the integral distribution function $F(x)$:

$$F(x)=\begin{cases}0, & x\le 1\\ \dfrac{x^2}{2}-\dfrac{x}{2}, & 1<x\le 2.\\ 1, & x>2\end{cases}$$

Example 2.3. The integral distribution function of a continuous variable X is given:

$$F(X)=\begin{cases}0, & x\le 0{,}5\\ \dfrac{2x-1}{2}, & 0{,}5<x\le 1{,}5\\ 1, & x>1{,}5\end{cases}.$$

Find the density function $f(x)$.

Solution. It is known that $f(x)=F'(x)$. Thus

$$f(x)=F'(X)=\begin{cases}(0)', & x\le 0{,}5\\ \left(\dfrac{2x-1}{2}\right)', & 0{,}5<x\le 1{,}5\\ (1)', & x>1{,}5\end{cases}=\begin{cases}0, & x\le 0{,}5\\ \dfrac{2-0}{2}, & 0{,}5<x\le 1{,}5\\ 0, & x>1{,}5\end{cases}=$$

$$=\begin{cases}0, & x\le 0{,}5\\ 1, & 0{,}5<x\le 1{,}5\\ 0, & x>1{,}5\end{cases}.$$

2.1. Numerical characteristics of continuous random variables

1) *Mathematical expectation* $M(X)$ is

$$M(X)=\int_{-\infty}^{\infty}x\cdot f(x)dx.$$

2) *Variance* $D(X)$ is

$$D(X)=\int_{-\infty}^{\infty}x^2\cdot f(x)dx-\left[M(X)\right]^2 \text{ or } D(X)=\int_{-\infty}^{\infty}\left(x-M(X)\right)^2\cdot f(x)dx.$$

3) *Root-mean-square deviation (or standard deviation)* $\sigma(X)$ is

$$\sigma(X)=\sqrt{D(X)}.$$

Example 2.4. The density function of a continuous variable X is given:

$$f(x)=\begin{cases}0, & x\le 0{,}5\\ 1, & 0{,}5<x\le 1{,}5\\ 0, & x>1{,}5\end{cases}.$$

Calculate: a) the mathematical expectation $M(X)$ and the variance $D(X)$; b) the probability that a random variable X lies in the interval $(1.0, 2.3)$; c) plot the graphs of the functions $F(X)$ and $f(x)$.

Solution. a) Let's find the mathematical expectation:

$$M(X)=\int_{-\infty}^{\infty} x\cdot f(x)dx=\int_{-\infty}^{0.5} x\cdot 0dx+\int_{0.5}^{1.5} x\cdot 1dx+\int_{1.5}^{\infty} x\cdot 0dx=$$

$$=0+\int_{0.5}^{1.5} xdx+0=\frac{x^2}{2}\Bigg|_{0.5}^{1.5}=\frac{1}{2}\cdot\left[1.5^2-0.5^2\right]=\frac{1}{2}\cdot[2.25-0.25]=\frac{1}{2}\cdot 2=1.$$

Let's calculate the variance:

$$D(X)=\int_{-\infty}^{\infty} x^2\cdot f(x)dx-\left[M(X)\right]^2=$$

$$=\int_{-\infty}^{0.5} x^2\cdot 0dx+\int_{0.5}^{1.5} x^2\cdot 1dx+\int_{1.5}^{\infty} x^2\cdot 0dx-1^2=0+\int_{0.5}^{1.5} x^2dx+0-1=$$

$$=\frac{x^3}{3}\Bigg|_{0.5}^{1.5}-1=\frac{1}{3}\cdot\left[1.5^3-0.5^3\right]-1=\frac{1}{3}\cdot[3.375-0.125]-1=$$

$$=\frac{1}{3}\cdot 3.25-1\approx 0.0833.$$

b) Let's find the probability that a random variable X lies in the interval $(1.0, 2.3)$ by the formula $P(x_1<X<x_2)=F(x_2)-F(x_1)$.

Thus,

$$P(1<X<2.3)=F(2.3)-F(1)=1-\frac{2\cdot 1-1}{2}=1-\frac{1}{2}=\frac{1}{2}=0.5.$$

c) Let's plot the graphs of the functions $F(X)$ and $f(x)$ (Fig. 2.5, 2.6).

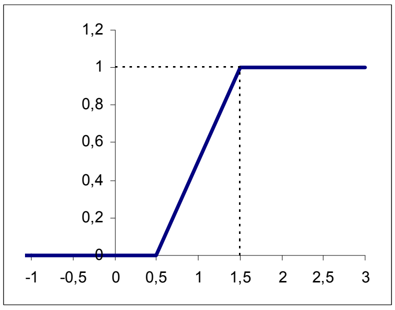

Fig. 2.5. **The graph of** $F(X)$

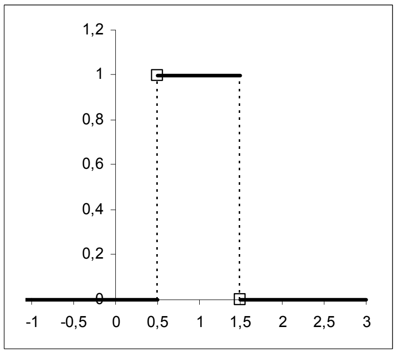

Fig. 2.6. **The graph of** $f(x)$

2.2. Main continuous distributions

2.2.1. Uniform distribution law. The uniform law of distribution is characterized by two functions: a probability density function $f(x)$ (the differential distribution function) (Fig. 2.1) and a cumulative distribution function $F(X)$ (the integral distribution function) (Fig. 2.2).

$$f(x)=\begin{cases}0, & x\le a\\ \dfrac{1}{b-a}, & a<x\le b,\\ 0, & x>b\end{cases}\qquad F(x)=\begin{cases}0, & x\le a\\ \dfrac{x-a}{b-a}, & a<x\le b.\\ 1, & x>b\end{cases}$$

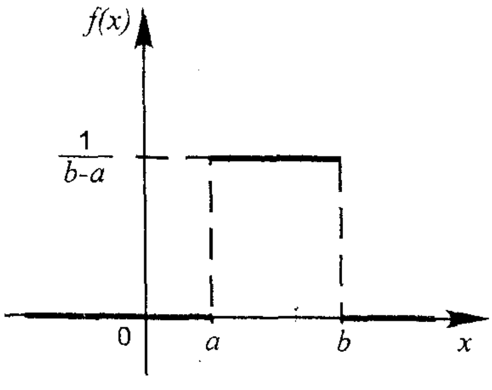

Fig. 2.1. **The graph of** $f(x)$

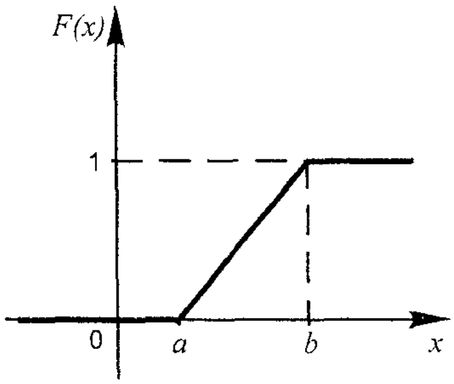

Fig. 2.2. **The graph of** $F(X)$

The probability that a random variable X lies in the interval (α,β) is equal to the increment of its integral distribution function on this interval; i.e.

$$P(\alpha<X<\beta)=F(\beta)-F(\alpha)=\frac{\beta-\alpha}{b-a}.$$

Numerical characteristics of uniform random variables are

1) *mathematical expectation* $M(X)$:

$$M(X)=\frac{a+b}{2};$$

2) *variance* $D(X)$:

$$D(X)=\frac{(b-a)^2}{12};$$

3) *root-mean-square deviation (or standard deviation)* $\sigma(X)$:

$$\sigma(X)=\frac{b-a}{2\sqrt{3}}.$$

Example 2.5. The parameters a,b of the uniform law of distribution are given: $a=2$ and $b=6$. Find: a) functions $f(x)$ and $F(x)$; b) the mathematical expectation $M(X)$, the variance $D(X)$ and the root-mean-square deviation $\sigma(X)$; c) $P(0<X<3)$.

Solution. Let's find functions $f(x)$ and $F(x)$ substituting $a=2$ and $b=6$ into formulas for functions:

$$f(x)=\begin{cases}0, & x\le 2\\ \dfrac{1}{6-2}, & 2<x\le 6\\ 0, & x>6\end{cases}=\begin{cases}0, & x\le 2\\ \dfrac{1}{4}, & 2<x\le 6,\\ 0, & x>6\end{cases}$$

$$F(x)=\begin{cases}0, & x\le 2\\ \dfrac{x-2}{6-2}, & 2<x\le 6\\ 1, & x>6\end{cases}=\begin{cases}0, & x\le 2\\ \dfrac{x-2}{4}, & 2<x\le 6.\\ 1, & x>6\end{cases}$$

Let's calculate the numerical characteristics $M(X)$, $D(X)$ and $\sigma(X)$:

$$M(X)=\frac{a+b}{2}=\frac{2+6}{2}=4, \qquad D(X)=\frac{(6-2)^2}{12}=\frac{4^2}{12}=\frac{4}{3},$$

$$\sigma(X) = \frac{b-a}{2\sqrt{3}} = \frac{6-2}{2\sqrt{3}} = \frac{4}{2\sqrt{3}} = \frac{2}{\sqrt{3}}.$$

Let's calculate the probability that a uniform random variable X lies in the interval $(0,3)$:

$$P(\alpha < X < \beta) = P(0 < X < 3) = F(3) - F(0) = \frac{3-0}{6-2} = \frac{3}{4} = 0.75.$$

2.2.2. Exponential distribution law. The exponential law of distribution is characterized by two functions: a probability density function $f(x)$ (the differential distribution function) (Fig. 2.3) and a cumulative distribution function $F(X)$ (the integral distribution function) (Fig. 2.4).

$$f(x) = \begin{cases} 0, & x < 0 \\ \lambda \cdot e^{-\lambda x}, & x \ge 0 \end{cases}, \qquad F(x) = \begin{cases} 0, & x < 0 \\ 1 - e^{-\lambda x}, & x \ge 0 \end{cases}.$$

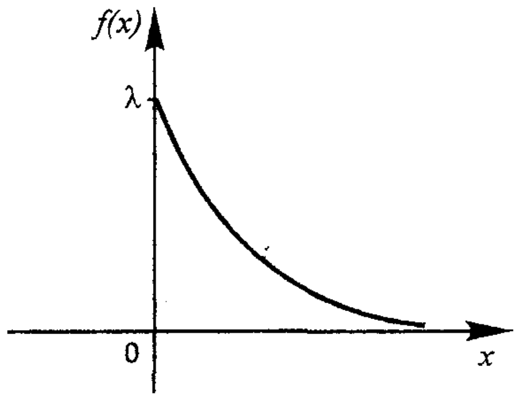

Fig. 2.3. **The graph of** $f(x)$

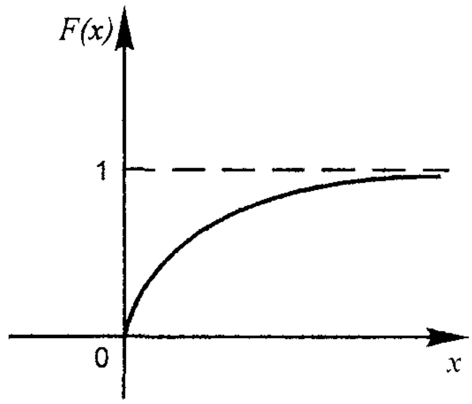

Fig. 2.4. **The graph of** $F(X)$

The probability that a random variable X lies in the interval (α, β) is equal to the increment of its integral distribution function on this interval; i.e.

$$P(\alpha < X < \beta) = F(\beta) - F(\alpha) = e^{-\lambda\alpha} - e^{-\lambda\beta}.$$

Numerical characteristics of exponential random variables are

1) *mathematical expectation* $M(X)$:

$$M(X)=\frac{1}{\lambda}.$$

2) *variance* $D(X)$:

$$D(X)=\frac{1}{\lambda^2},$$

3) *root-mean-square deviation (or standard deviation)* $\sigma(X)$:

$$\sigma(X)=\frac{1}{\lambda}.$$

Example 2.6. The probability density function of the exponential law of distribution $f(x)=\begin{cases}0, & x<0\\ 0.05\cdot e^{-0.05x}, & x\ge 0\end{cases}$ is given. Find: a) the function $F(x)$; b) the mathematical expectation $M(X)$, the variance $D(X)$ and the root-mean-square deviation $\sigma(X)$; c) $P(2<X<10)$.

Solution. We have that $\lambda=0.05$ from the formula of $f(x)$. Let's substitute this parameter into the formula for $F(x)$ and find the numerical characteristics:

$$F(x)=\begin{cases}0, & x<0\\ 1-e^{-\lambda x}, & x\ge 0\end{cases}=\begin{cases}0, & x<0\\ 1-e^{-0.05x}, & x\ge 0\end{cases},$$

$$M(X)=\frac{1}{\lambda}=\frac{1}{0.05}=20,\ D(X)=\frac{1}{\lambda^2}=\frac{1}{0.05^2}=400,\ \sigma(X)=\frac{1}{\lambda}=\frac{1}{0.05}=20.$$

Let's calculate the probability $P(2<X<10)$ that a random variable X lies in the interval $(2,10)$:

$$P(\alpha<X<\beta)=P(2<X<10)=e^{-\lambda\alpha}-e^{-\lambda\beta}=e^{-0.05\cdot 2}-e^{-0.05\cdot 10}=$$
$$=0.90484-0.60653=0.29831.$$

2.2.3. Normal law distribution. The normal law of distribution is characterized by two functions: a probability density function $f(X)$ (the differential distribution function) (Fig. 2.5) and a cumulative distribution function $F(X)$ (the integral distribution function) (Fig. 2.6)

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(x-a)^2}{2\sigma^2}} = \frac{\varphi(t)}{\sigma}$$

$$F(x) = \int_{-\infty}^{x} \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(x-a)^2}{2\sigma^2}} dx = \frac{1}{2} + \Phi(t),$$

where $\varphi(t) = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{t^2}{2}}$ and $\Phi(t) = \frac{1}{\sqrt{2\pi}} \int_0^t \cdot e^{-\frac{t^2}{2}} dt$ are Laplace differential and integral functions (see Table 1), $t = \frac{x-a}{\sigma}$ is a normalized value

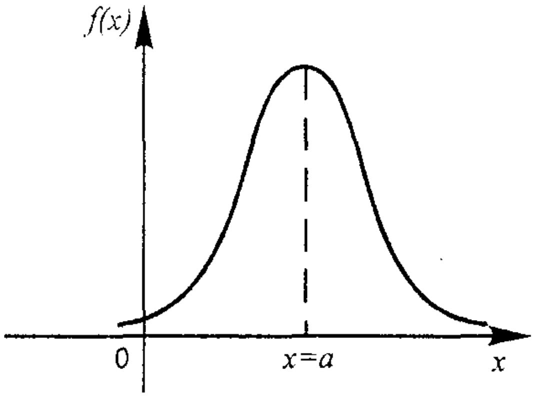

Fig. 2.5. **A density curve $f(x)$ graph**

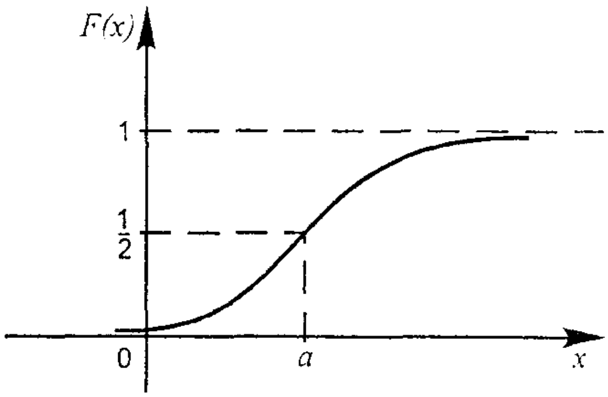

Fig. 2.6. **The graph of $F(X)$**

The probability that a random variable X lies in the interval (α, β) is equal to the increment of its integral distribution function on this interval; i.e.

$$P(\alpha < X < \beta) = F(\beta) - F(\alpha) = \Phi\left(\frac{\beta - a}{\sigma}\right) - \Phi\left(\frac{\alpha - a}{\sigma}\right).$$

Numerical characteristics of exponential random variables are

1) *mathematical expectation* $M(X)$:

$$M(X) = a \qquad (M(t) = 0),$$

2) *variance* $D(X)$:

$$D(X)=\sigma^2 \qquad (D(t)=1),$$

3) *root-mean-square deviation (or standard deviation)* $\sigma(X)$:

$$\sigma(X)=\sigma \qquad (\sigma(t)=1).$$

Example 2.7. The probability density function of the normal law of distribution $f(x)=\frac{1}{2\sqrt{2\pi}}\cdot e^{-\frac{(x-3)^2}{8}}$ is given. Find the integral function, calculate the numerical characteristics and $P(1<X<7)$.

Solution. We have that $a=3$ and $\sigma=2$ from the formula of $f(x)$. Let's substitute these parameters into the formula for $F(x)$ and find the numerical characteristics:

$$F(x)=\int_{-\infty}^{x}\frac{1}{\sigma\sqrt{2\pi}}\cdot e^{-\frac{(x-a)^2}{2\sigma^2}}dx=\int_{-\infty}^{x}\frac{1}{2\sqrt{2\pi}}\cdot e^{-\frac{(x-3)^2}{2\cdot 2^2}}dx=$$

$$=\int_{-\infty}^{x}\frac{1}{2\sqrt{2\pi}}\cdot e^{-\frac{(x-3)^2}{8}}dx,$$

$M(X)=a=3$, $D(X)=\sigma^2=2^2=4$ and $\sigma(X)=\sigma=2$.

Let's calculate the probability $P(1<X<7)$ that a random variable X lies in the interval $(1,7)$:

$$P(\alpha<X<\beta)=P(1<X<7)=\Phi\left(\frac{\beta-a}{\sigma}\right)-\Phi\left(\frac{\alpha-a}{\sigma}\right)=$$

$$=\Phi\left(\frac{7-3}{2}\right)-\Phi\left(\frac{1-3}{2}\right)=\Phi(2)-\Phi(-1)=\left\|\begin{matrix}\textit{use the property}\\ \Phi(-x)=\Phi(x)\end{matrix}\right\|=\Phi(2)+\Phi(1)=$$

$$=\left\|\begin{matrix}\textit{use table } 1\\ \Phi(2)=0.4772\\ \Phi(1)=0.3413\end{matrix}\right\|=0.4772-0.3413=0.1359.$$

Control tasks. Discrete random variables

1.1. Statisticians use sampling plans to either accept or reject batches or lots of material. Suppose one of these sampling plans involves sampling independently 10 items from a lot of 100 items in which 12 are defective. Make up a distribution law of the number of items found defective in the sample of 10 (in this case, the random variable takes on the values 0, 1, 2, ..., 9, 10).

1.2. The factory sent 5 000 products of high quality to the warehouse. The probability of damaging the products on the way is equal to 0.0004. Make up a distribution law of a number of damaged products received at the warehouse. Find the probability that a) from 2 to 3 damaged products will be received at the warehouse; b) at least one damaged product will be received at the warehouse. Calculate the numerical characteristics $M(X)$, $D(X)$ and $\sigma(X)$. Find a distribution function of a random variable X.

1.3. There are 500 defective details The probability of producing defective details is equal to 0.01. Make up a distribution law of a number of defective details. Find the probability that there will be: a) 3 defective details; b) from 2 to 4 defective ones.

1.4. A coin is tossed 5 times. Make up a distribution law of a number of occurrences of heads. Find the probability that the coin will land with a head at a least 2 times. Calculate $M(X)$, $D(X)$ and $\sigma(X)$. Find a distribution function of a random variable X.

1.5. The probability of train arrival at the station on time is equal to 0.8. Make up a distribution law of a number of train arrival at the station on time out of 4 expecting trains. Find the probability that no less than 2 and no more than 3 trains will arrive on time. Find the probability that at least one train will arrive on time. Calculate $M(X)$, $D(X)$ and $\sigma(X)$. Find a distribution function of a random variable X.

1.6. The probability of the birth of a boy is equal to 0.51. Make up a distribution law of a number of newborn boys out of 10 newborns. Find the probability that among 10 newborns there will be from 3 to 7 boys. Calculate $M(X)$, $D(X)$ and $\sigma(X)$. Find a distribution function of a random variable X.

1.7. The probability that the student will pass a test from the very first time is equal to 0.9. Make up a distribution law of a number of the students who will pass a test from the very first among 7 students of the same knowledge level. Find the probability that from 4 to 6 students will pass a test.

1.8. There is a random variable X

a)

x_i	5	10	15	20
p_i	0,2	0,3	0,4	0,1

b)

x_i	0	1	2	3	4
p_i	0,1	0,25	0,35	0,2	?

c)

x_i	-1	1	3	5	7
p_i	0,10	0,19	0,31	0,25	0,15

d)

x_i	1	2	3	4	5
p_i	0,2	?	0,1	0,3	0,1

Draw a distribution polygon. Calculate $M(X)$, $D(X)$ and $\sigma(X)$. Find a distribution function of a random variable X.

Control tasks. Continuous random variables

2.1. The integral distribution function of a continuous variable X is given. Find: a) the density function $f(x)$; b) the numerical characteristics $M(X)$, $D(X)$ and $\sigma(X)$; c) the probability $P(\alpha < X < \beta)$; d) plot the graphs of functions $F(X)$ and $f(x)$.

1)
$$F(X)=\begin{cases}0, & x\le -2\\ \dfrac{x+2}{4}, & -2<x\le 2\\ 1, & x>2\end{cases}$$
$\alpha=0{,}5$, $\beta=4{,}5$;

2)
$$F(X)=\begin{cases}0, & x\le 0{,}5\\ \dfrac{2x-1}{2}, & 0{,}5<x\le 1{,}5\\ 1, & x>1{,}5\end{cases}$$
$\alpha=1{,}0$, $\beta=2{,}3$;

3)
$$F(X)=\begin{cases}0, & x\le 1\\ \dfrac{1}{4}(x-1)^2, & 1<x\le 3\\ 1, & x>3\end{cases}$$
$\alpha=0{,}2$, $\beta=2{,}6$;

4)
$$F(X)=\begin{cases}0, & x\le 5\\ \dfrac{1}{4}(x-5)^2, & 5<x\le 7\\ 1, & x>7\end{cases}$$
$\alpha=3{,}4$, $\beta=6{,}6$.

2.2. The density function of a continuous variable X is given. Find: a) the density function $f(x)$; b) the numerical characteristics $M(X)$, $D(X)$, $\sigma(X)$; c) the probability $P(\alpha < X < \beta)$; d) plot graphs of $F(X)$ and $f(x)$.

1) $f(x)=\begin{cases}0, & x\le 1\\ \frac{x}{4}, & 1<x\le 3\\ 0, & x>3\end{cases}$

$\alpha=0$, $\beta=2$;

2) $f(x)=\begin{cases}0, & x\le 1\\ \frac{1}{2}(x-1), & 1<x\le 3\\ 0, & x>3\end{cases}$

$\alpha=1.5$, $\beta=5$;

3) $f(x)=\begin{cases}0, & x\le 0,\\ \frac{3x^2}{8}, & 0<x\le 2,\\ 0, & x>2.\end{cases}$

$\alpha=-1$, $\beta=1.5$;

4) $f(x)=\begin{cases}0, & x\le 3,\\ \frac{1}{25}(x-3)^2, & 3<x\le 8,\\ 0, & x>8.\end{cases}$

$\alpha=4$, $\beta=6$.

2.3. The density function of a continuous variable X is given:

1) $f(x)=\begin{cases}0, & x\le 0\\ cx^2, & 0<x\le 1,\\ 0, & x>1\end{cases}$

2) $f(x)=\begin{cases}0, & x\le 2\\ c(x-2), & 2<x\le 5.\\ 0, & x>5\end{cases}$

Find: a) the parameter c; b) the integral distribution function $F(x)$; c) the probability $P(0.5<X<3)$; d) the numerical characteristics $M(X)$, $D(X)$ and $\sigma(X)$. Plot the graphs of the functions $f(x)$ and $F(x)$;

2.4. The density function of the exponential distribution $f(x)=\begin{cases}0, & x<0\\ 0.04\cdot e^{-0.04x}, & x\ge 0\end{cases}$ is given. Find: a) the function $F(x)$; b) $M(X)$, $D(X)$ and $\sigma(X)$; c) $P(1<X<2)$.

2.5. The waiting time, in hours, between successive speeders spotted by a radar unit is an exponential continuous random variable with cumulative distribution function $F(x)=\begin{cases}0, & x<0\\ 1-e^{-8x}, & x\ge 0\end{cases}$. Find: a) the density function $f(x)$; b) $M(X)$, $D(X)$ and $\sigma(X)$; c) the probability of waiting less than 12 minutes between successive speeders.

2.6. The mathematical expectation of a normal random variable X equals 20, the root-mean-square deviation equals 5. Find: a) functions $f(x)$

and $F(x)$; b) the numerical characteristics $M(X)$, $D(X)$ and $\sigma(X)$; c) the probability that a random variable X lies in the interval $(15, 25)$.

2.7. The mathematical expectation of a normal random variable X equals 25. It is known that $P(35 < X < 40) = 0{,}2$. Find the probability $P(10 < X < 15)$.

2.8. The parameters a, b of the uniform law of distribution are given: $a = 1$ and $b = 4$. Find: a) the functions $f(x)$ and $F(x)$; b) $M(X)$, $D(X)$ and $\sigma(X)$; c) $P(0 < X < 3)$, $P(2 < X < 5)$.

2.9. A man's height is distributed by the normal law. The mathematical expectation of a normal random variable X equals 170 centimeters, the root-mean-square deviation equals 5 centimeters. Find the probability that the men' height there will be a) less than 160 centimeters; b) greater than 180 centimeters; c) from 160 to 175 centimeters.

2.10. The proportion of people who respond to a certain mail-order solicitation is a continuous random variable X that has the density function $f(x) = \begin{cases} 0, & x \le 0 \\ \frac{2(x+2)}{4}, & 0 < x \le 1 \\ 0, & x > 1 \end{cases}$. Show that $P(0 < X < 1) = 1$. Find the probability that more than 1/4 but fewer than 1/2 of the people contacted will respond to this type of solicitation.

2.11. Measurements of scientific systems are always subject to variation, some more than others. There are many structures for measurement error and statisticians spend a great deal of time modeling these errors. Suppose the measurement error X of a certain physical quantity is decided by the density function $f(x) = \begin{cases} 0, & x \le -1 \\ c\left(3 - x^2\right), & -1 < x \le 1 \\ 0, & x > 1 \end{cases}$. Determine c that renders $f(x)$ a valid density function. Find the probability that a random error in measurement is less than 1/2. For this particular measurement, it is undesirable if the magnitude of the error, exceeds 0.8. What is the probability that this occurs?

Theoretical questions

1. What do you call a random variable?
2. Give a definition of a discrete random variable.
3. What is a density function of a discrete random variable?
4. Give a definition of a distribution law.
5. What do you call a distribution polygon?
6. The numerical characteristics of a distribution (a mathematical expectation, a variance, a root-mean-square deviation) and their properties.
7. Give a definition of a distribution function and write down its properties.
8. Write down the formula of the probability that a discrete random variable X lies in the interval (x_1, x_2).
9. Binomial distribution law and its numerical characteristics.
10. Geometric distribution law and its numerical characteristics.
11. Hypergeometric distribution law and its numerical characteristics.
12. Poisson distribution law and its numerical characteristics.
13. Give a definition of a continuous random variable.
14. Give a definition of a cumulative distribution function and its properties.
15. Give a definition of a distribution density and its properties.
16. Write down the condition of normalization of a density function.
17. The numerical characteristics of a distribution (a mathematical expectation, a variance, a root-mean-square deviation) and their properties.
18. Write down the formula of the probability that a continuous random variable X lies in the interval (x_1, x_2).
19. A uniform distribution law and its numerical characteristics.
20. An exponential distribution law and its numerical characteristics.
21. A normal distribution law and its standard representation.
22. Classify the following random variables as discrete or continuous:

a) the number of automobile accidents per year in the city;
b) the length of time to play 18 holes of golf;
c) the amount of milk produced yearly by a particular cow;
d) the number of eggs laid each month by a hen;
e) the number of building permits issued each month in a certain city;
f) the weight of grain produced per acre.

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Appendix A

Table 1

Values of Laplace cumulative distribution function $\Phi(t) = \frac{1}{\sqrt{2\pi}} \int_0^t e^{-\frac{z^2}{2}} dz$

t	0	1	2	3	4	5	6	7	8	9
0,0	0,0000	0,0040	0,0080	0,0120	0,0160	0,0199	0,0239	0,0279	0,0319	0,0359
0,1	0,0398	0,0438	0,0478	0,0517	0,0557	0,0596	0,0636	0,0675	0,0714	0,0754
0,2	0,0793	0,0832	0,0871	0,0910	0,0948	0,0987	0,1026	0,1064	0,1103	0,1141
0,3	0,1179	0,1217	0,1255	0,1293	0,1331	0,1368	0,1406	0,1443	0,1480	0,1517
0,4	0,1554	0,1591	0,1628	0,1664	0,1700	0,1736	0,1772	0,1808	0,1844	0,1879
0,5	0,1915	0,1950	0,1985	0,2019	0,2054	0,2088	0,2123	0,2157	0,2190	0,2224
0,6	0,2258	0,2291	0,2324	0,2356	0,2389	0,2422	0,2454	0,2486	0,2518	0,2549
0,7	0,2580	0,2612	0,2642	0,2673	0,2704	0,2734	0,2764	0,2794	0,2823	0,2852
0,8	0,2881	0,2910	0,2939	0,2967	0,2996	0,3023	0,3051	0,3078	0,3106	0,3133
0,9	0,3159	0,3186	0,3212	0,3238	0,3264	0,3289	0,3315	0,3340	0,3365	0,3389
1,0	0,3413	0,3438	0,3461	0,3485	0,3508	0,3531	0,3554	0,3577	0,3599	0,3621
1,1	0,3643	0,3665	0,3686	0,3708	0,3729	0,3749	0,3770	0,3790	0,3810	0,3830
1,2	0,3849	0,3869	0,3888	0,3906	0,3925	0,3944	0,3962	0,3980	0,3997	0,4015
1,3	0,4032	0,1049	0,4066	0,4082	0,4099	0,4115	0,4131	0,4147	0,4162	0,4177
1,4	0,4192	0,4207	0,4222	0,4236	0,4251	0,4265	0,4274	0,4292	0,4306	0,4319
1,5	0,4332	0,4345	0,4357	0,4370	0,4382	0,4394	0,4406	0,4418	0,4430	0,4441
1,6	0,4452	0,4463	0,4474	0,4484	0,4495	0,4505	0,4515	0,4525	0,4535	0,4545
1,7	0,4554	0,4564	0,4573	0,4582	0,4591	0,4599	0,4608	0,4616	0,4625	0,4633
1,8	0,4641	0,4648	0,4656	0,4664	0,4671	0,4678	0,4686	0,4693	0,4700	0,4706
1,9	0,4713	0,4719	0,4726	0,4732	0,4738	0,4744	0,4750	0,4756	0,4762	0,4757
2,0	0,4772	0,4778	0,4783	0,4788	0,4796	0,4798	0,4803	0,4808	0,48Ï2	0,4817
2,1	0,4821	0,4826	0,4830	0,4834	0,4838	0,4842	0,4846	0,4850	0,4854	0,4857
2,2	0,4861	0,4864	0,4868	0,4871	0,4874	0,4878	0,4881	0,4884	0,4887	0,4890
2,3	0,4893	0,4896	0,4898	0,4901	0,4903	0,4906	0,4909	0,4911	0,4913	0,4916
2,4	0,4918	0,4920	0,4922	0,4924	0,4927	0,4929	0,4930	0,1932	0,4934	0,4936
2,5	0,4938	0,4940	0,4941	0,4943	0,4945	0,4946	0,4948	0,4949	0,4951	0,4952
2,6	0,4953	0,4955	0,4956	0,4957	0,4958	0,4960	0,4961	0,4962	0,4963	0,4964
2,7	0,4965	0,4966	0,4967	0,4968	0,4969	0,4970	0,4971	0,4972	0,4973	0,4973
2,8	0,4974	0,4975	0,4976	0,4977	0,4977	0,4978	0,4979	0,4980	0,4980	0,4981
2,9	0,4981	0,4982	0,4982	0,4983	0,4984	0,4984	0,4985	0,4985	0,4986	0,4986
3,0	0,4986	0,4986	0,4987	0,4987	0,4988	0,4988	0,4988	0,4989	0,4989	0,4990
3,1	0,4990	0,4990	0,4991	0,4991	0,4991	0,4992	0,4992	0,4992	0,4992	0,4993
3,2	0,4993	0,4993	0,4993	0,4994	0,4994	0,4994	0,4994	0,4994	0,4995	0,4995
3,3	0,4995	0,4995	0,4995	0,4996	0,4996	0,4996	0,4996	0,4996	0,4997	0,4997
3,4	0,4997	0,4997	0,4997	0,4997	0,4997	0,4998	0,4998	0,4998	0,4998	0,4998
3,5	0,4998	0,4998	0,4998	0,4998	0,4998	0,4998	0,4998	0,4998	0,4998	0,4998
3,6	0,4998	0,4998	0,4998	0,4998	0,4998	0,4998	0,4999	0,4999	0,4999	0,4999
3,7	0,4999	0,4999	0,4999	0,4999	0,4999	0,4999	0,4999	0,4999	0,4999	0,4999
3,8	0,4999	0,4999	0,4999	0,4999	0,4999	0,5000	0,5000	0,5000	0,5000	0,5000
3,9	0,5000	0,5000	0,5000	0,5000	0,5000	0,5000	0,5000	0,5000	0,5000	0,5000

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